A preliminary study on some *a posteriori* error estimates for soft-tissue biomechanics

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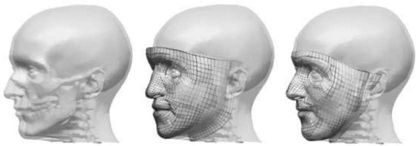
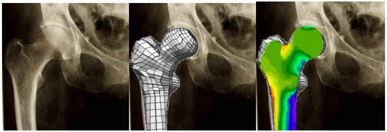
Patient-specific finite element models of soft-tissues

- Design of personalized medical devices
- Computer-assisted planning or guidance



Bucki, Lobos, Payan & Hitschfeld 2011



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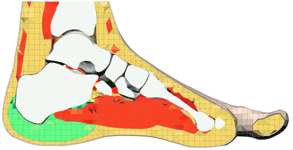
Meshing is still a big challenge

- Complex geometries
- Geometric quality of the elements ?
- Impact on the accuracy of the results ?
  - ➥ **discretization error**



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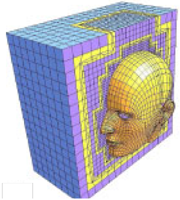
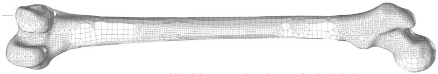
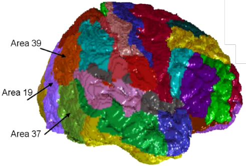


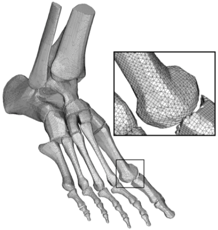
Zhang, Hughes & Bajaj 2010



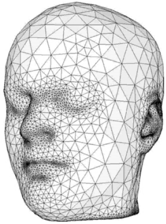
Ito Y, Shih A & Soni 2009



Ebeida, Patney, Owens & Mestreau 2011

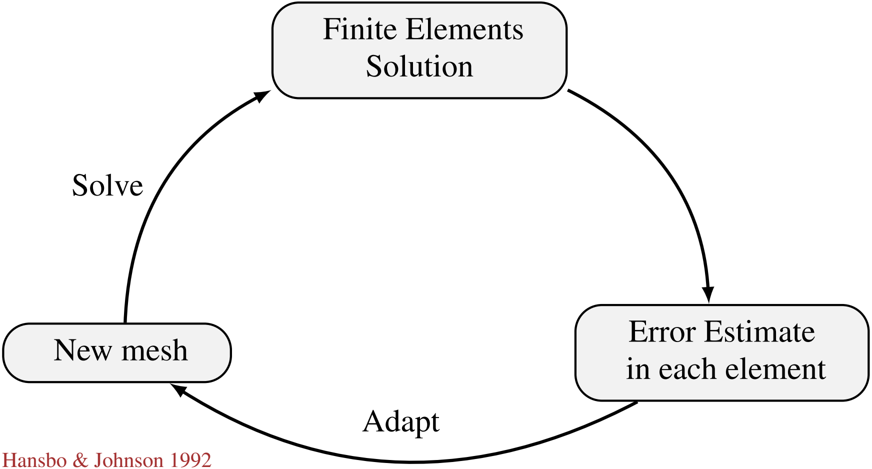


Lobos & Gonzalez 2015



Frey & Georges 2008

A posteriori error estimate : Optimal / Economical mesh



Hansbo & Johnson 1992
Ainsworth & Oden 1997
Verfürth 1999

Simplified setting for a preliminary study

2D linear elasticity with (possibly) linearized fiber activation :

$$\min_{\mathbf{u}\in V} \int_\Omega \boldsymbol{\sigma}(\mathbf{u}) : \boldsymbol{\varepsilon}(\mathbf{u})$$

where $\boldsymbol{\sigma}(\mathbf{u}) = \underbrace{\boldsymbol{\sigma}_P(\mathbf{u})}_{\text{passive part}} + \underbrace{\boldsymbol{\sigma}_A}_{\text{fiber activation}}$ with $\begin{cases} \boldsymbol{\sigma}_A = \beta T e_A \otimes e_A \\ e_A : \text{ fiber direction} \\ T : \text{ tension} \\ \beta : \text{ activation parameter} \end{cases}$

Cowin & Humphrey 2001
Payan & Ohayon 2017
Biomechanics of Living Organs : Hyperelastic Constitutive Laws for Finite Element Modeling.

Goal oriented error estimates

Variational setting (for linear problems) : $\mathbf{u} \in \mathbf{V}$ s.t.

$$a(\mathbf{u}, \mathbf{v}) = l(\mathbf{v}) \quad \forall\, \mathbf{v} \in \mathbf{V}$$

Here

$$a(\mathbf{u}, \mathbf{v}) = \int_{\Omega} \boldsymbol{\sigma}_P(\mathbf{u}) : \boldsymbol{\varepsilon}(\mathbf{u}) \quad \text{and} \quad l(\mathbf{v}) = \int_{\Omega} \boldsymbol{\sigma}_A : \boldsymbol{\varepsilon}(\mathbf{u})$$

Linear quantity of interest :

$$J : \mathbf{V} \ni \mathbf{u} \mapsto J(\mathbf{u}) \in \mathbb{R}.$$

Aim : $|J(\mathbf{u}) - J(\mathbf{u}_h)|$?

Dual Weighted Residuals (DWR) error estimates

Fundamental observation :

$$J(\mathbf{u}) - J(\mathbf{u}_h) = a(\mathbf{u}, \mathbf{z}) - a(\mathbf{u}_h, \mathbf{z}) = l(\mathbf{z}) - a(\mathbf{u}_h, \mathbf{z}) =: \eta_h(\mathbf{z}).$$

$$\text{with } \mathbf{z} \in \mathbf{V} \quad \text{s.t.} \quad a(\mathbf{v}, \mathbf{z}) = J(\mathbf{v}), \qquad \forall\, \mathbf{v} \in \mathbf{V}.$$

➥ **global estimator** : η_h

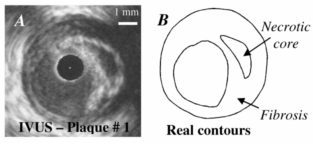
Moreover :

$$\begin{aligned} J(\mathbf{u}) - J(\mathbf{u}_h) &= \sum_{K \in \mathcal{K}_h} \langle R_K, \mathbf{z} - \pi_h \mathbf{z} \rangle_K + \langle R_{\partial K}, \mathbf{z} - \pi_h \mathbf{z} \rangle_{\partial K} \\ &=: \sum_{K \in \mathcal{K}_h} \eta_K(z - \pi_h z) \quad \textbf{Error by element} \end{aligned}$$

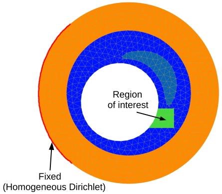
where $\pi_h \mathbf{z}$ is an interpolant and R_K and $R_{\partial K}$ are the residual contribution over the cell K and his boundary ∂K.

➥ **local estimators** : η_K

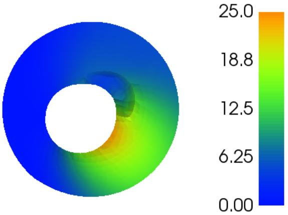
Becker & Rannacher 1997, 2001
Rognes & Logg 2013 (Generic implementation in FENICS)



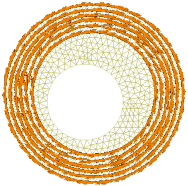
Le Floc'h et al 2008



Region of interest ω (green)



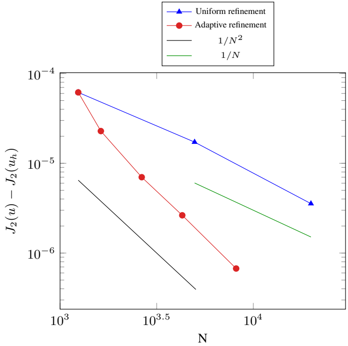
Displacement

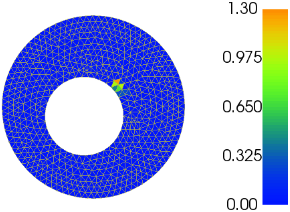


Activation

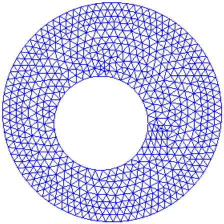
Quantity of interest : $J(\mathbf{u}) = \frac{1}{2}\int_{\omega} \operatorname{tr} \boldsymbol{\varepsilon}(\mathbf{u}) d\mathbf{x} \in \mathbb{R}$

1st Example : Artery with fibre activation

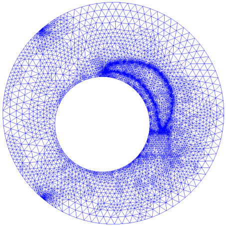




Initial error estimator η_K

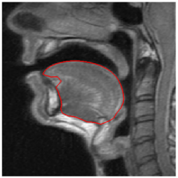


Initial mesh

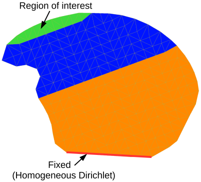


Adapted mesh (5th iteration)

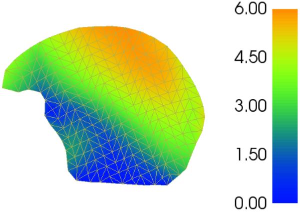
2nd Example : Tongue with fibre activation



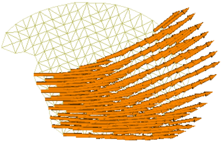
Bijar, Rohan, Perrier & Payan 2015



Region of interest ω (green)



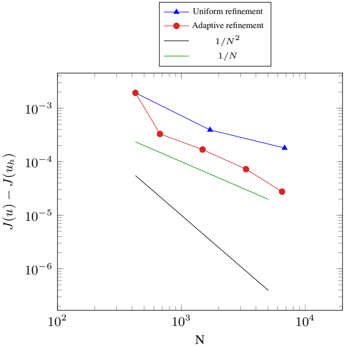
Displacement

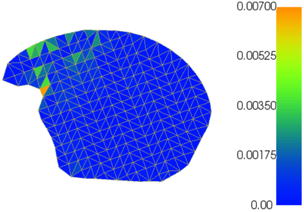


Activation of Genioglossus

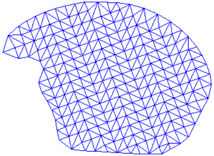
Quantity of interest : $J(\mathbf{u}) = \frac{1}{2}\int_\omega \mathrm{tr}\, \boldsymbol{\varepsilon}(\mathbf{u}) d\mathbf{x} \in \mathbb{R}$

2nd Example : Tongue with fibre activation

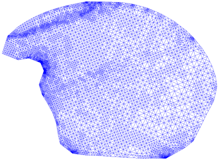




Initial error estimator η_K



Initial mesh



Adapted mesh (5th iteration)

Conclusion

Motivations

- *A posteriori* error estimators :
  Rarely used in practice for biomedical simulations
- Allow to quantify the discretization error
- Discretization error may have serious implications for guidance/design

Results

- DWR estimate : discretization error for a goal-oriented quantity
- Mesh adaptivity : optimal accuracy with low computational cost
- Promising preliminary results in a simplified setting

Perspectives

- Errors coming from the parameters and the model
- Hyperelasticity & 3D
- Other error estimator

Acknowledgement

For the discussions

- R. Becker
- F. Hubert
- J. Ohayon
- ...
- Y. Payan
- P. Perrier
- J. Hale

For their financing

REGION BOURGOGNE FRANCHE COMTE

Conclusion

A posteriori error estimators :

- Error per cell
- Optimal mesh
- Improvement of the simulations

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- Optimal mesh
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Happy Birthday Karol ! ! !