LES MATERIAUX

FONT-ILS

DES MATHEMATIQUES ?

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Approche interdisciplinaire mathématiques / physique en 2nde GT, 1ère S

PLAN

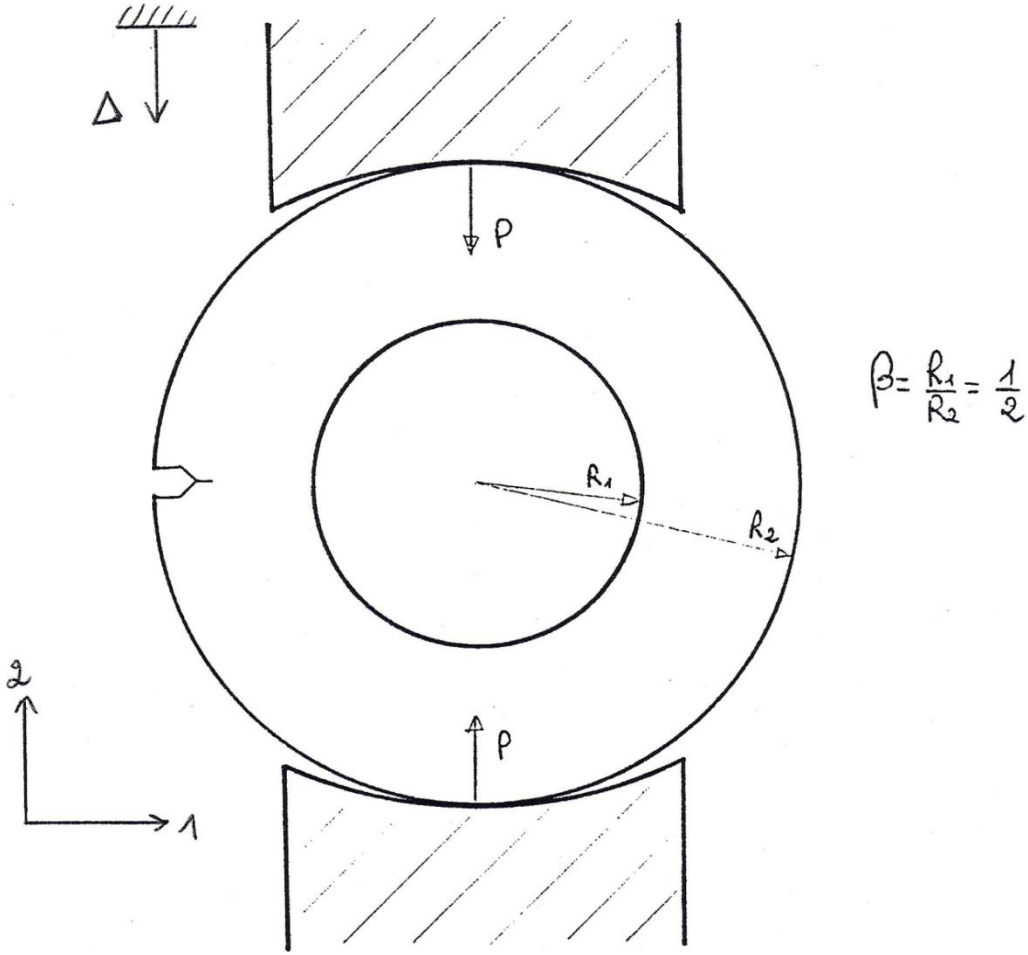
I) DISPOSITIF EXPERIMENTAL :
LA COURONNE DE PMMA

II) RESULTATS ET OBSERVATIONS :
ZOOM SUR LES FACIES DE RUPTURE …

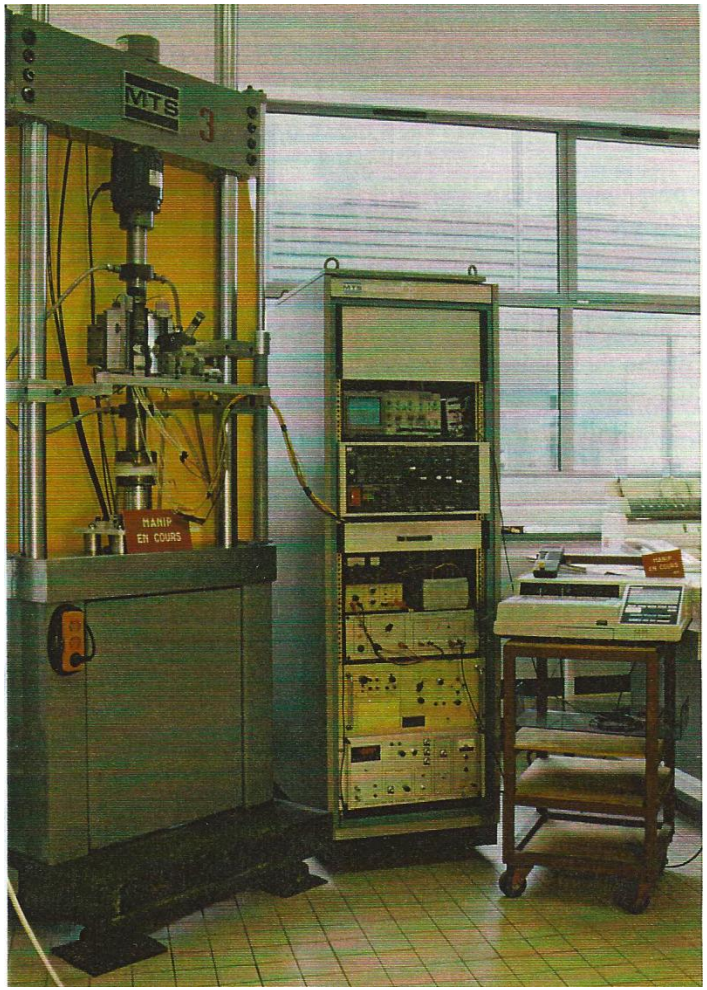
III) MODELISATION :
DES FRONTS DE FISSURE SECONDAIRES

IV) VALIDATION :
ENTRE THEORIE ET EXPERIENCE …

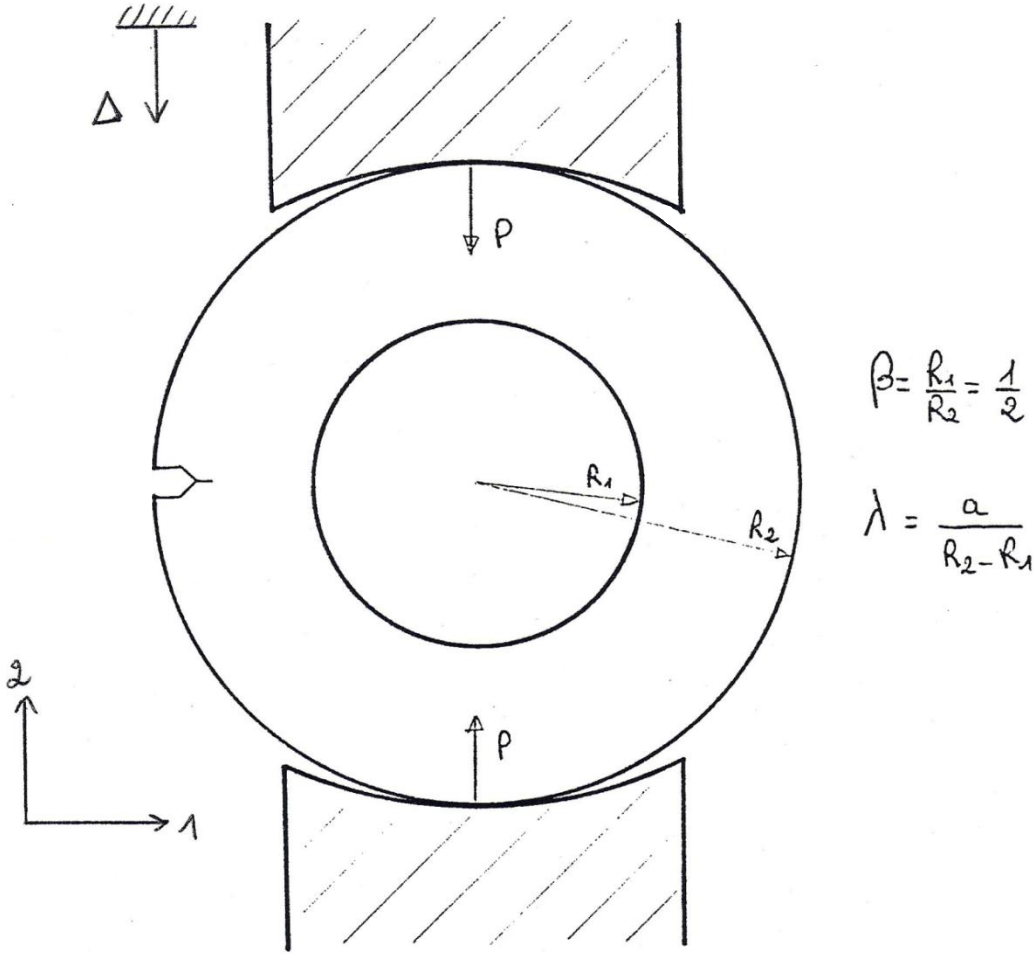
V) APPROCHE EN 3D :
REPRESENTATION SPATIO-TEMPORELLE …



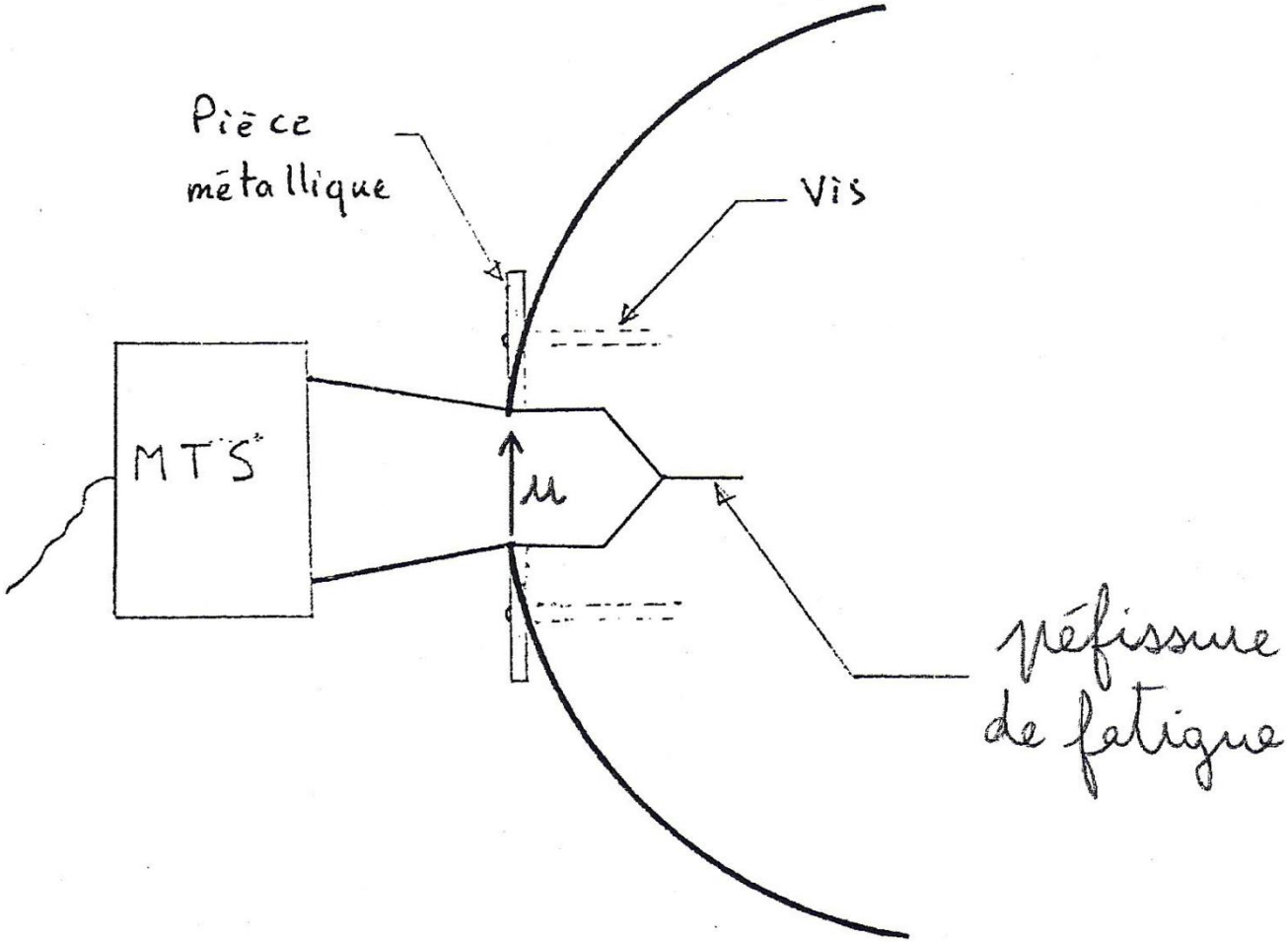
Dispositif expérimental



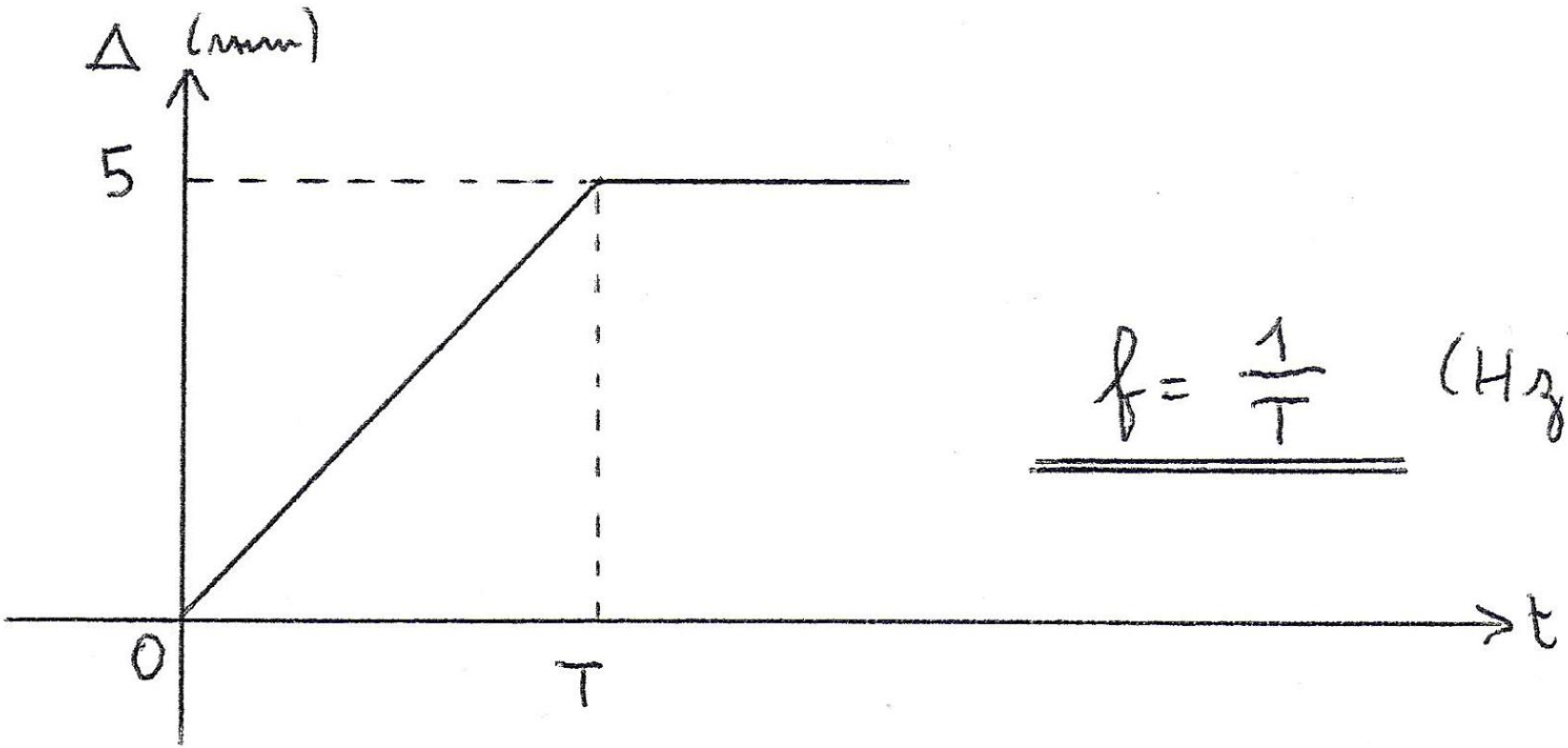
Machine d'essais servo-hydraulique 25 tonnes



Dispositif expérimental

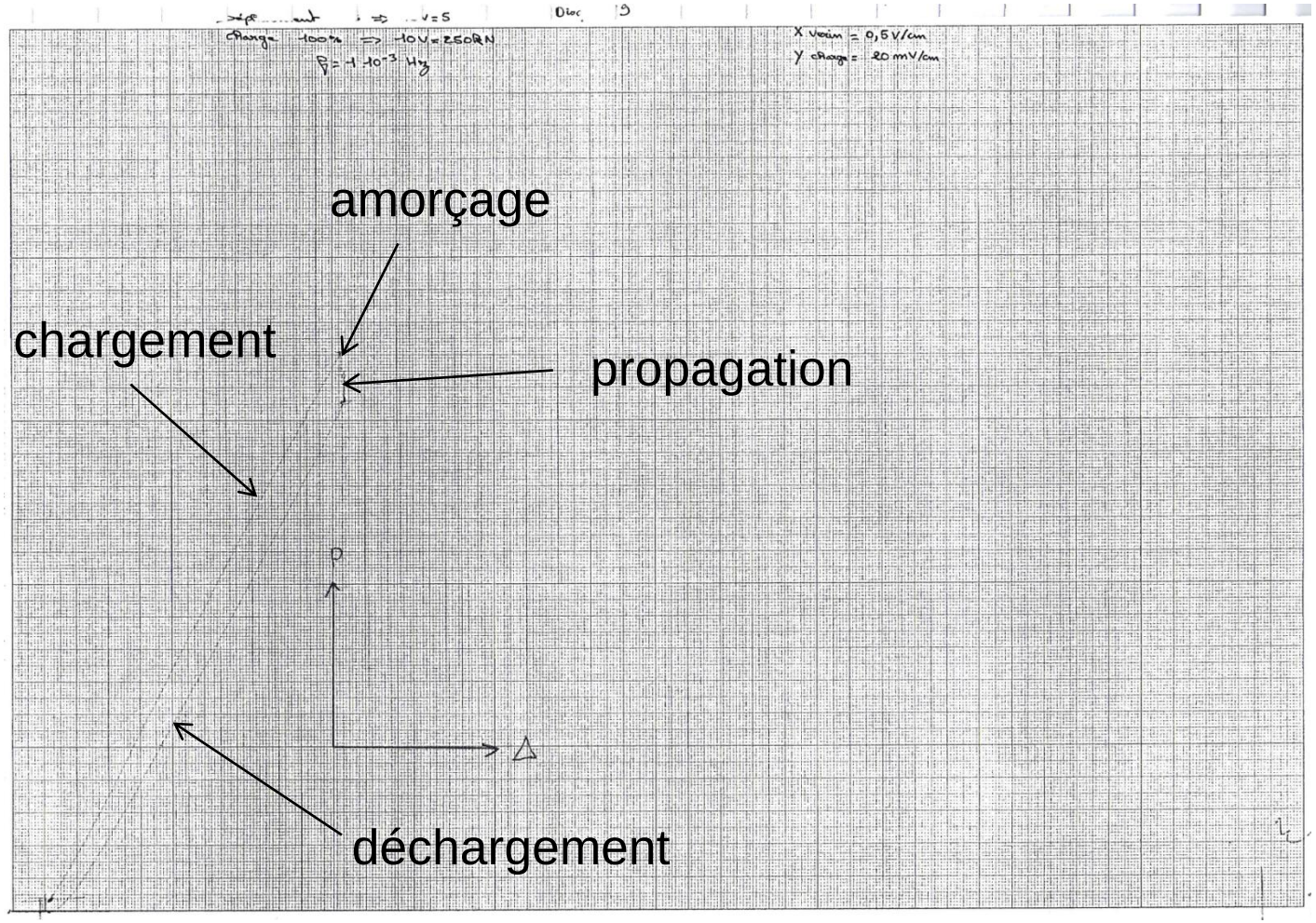


Mesure de l'ouverture de la fissure par extensométrie

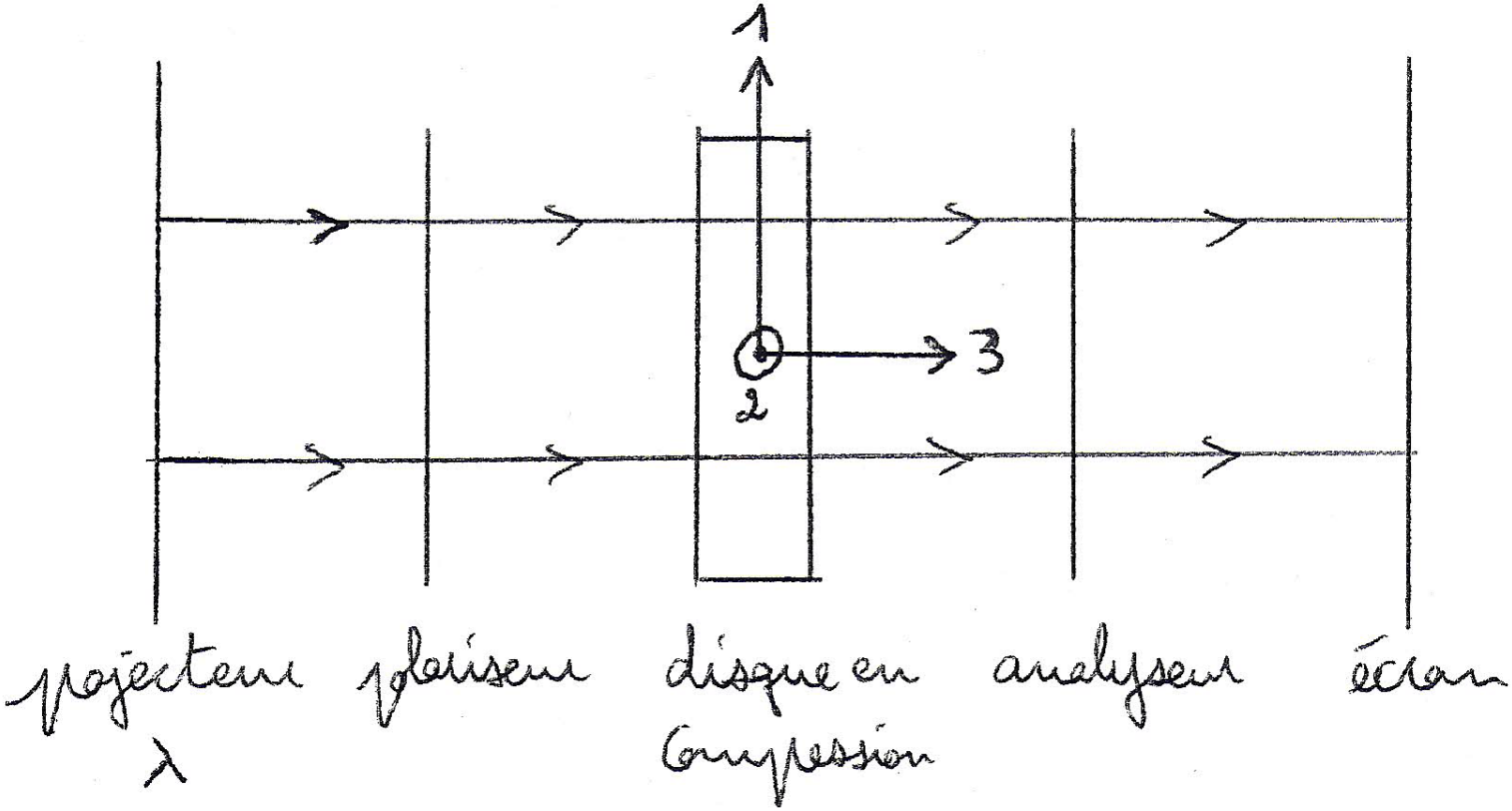


$$f = \frac{1}{T} \quad (Hz)$$

Echelon de déplacement imposé

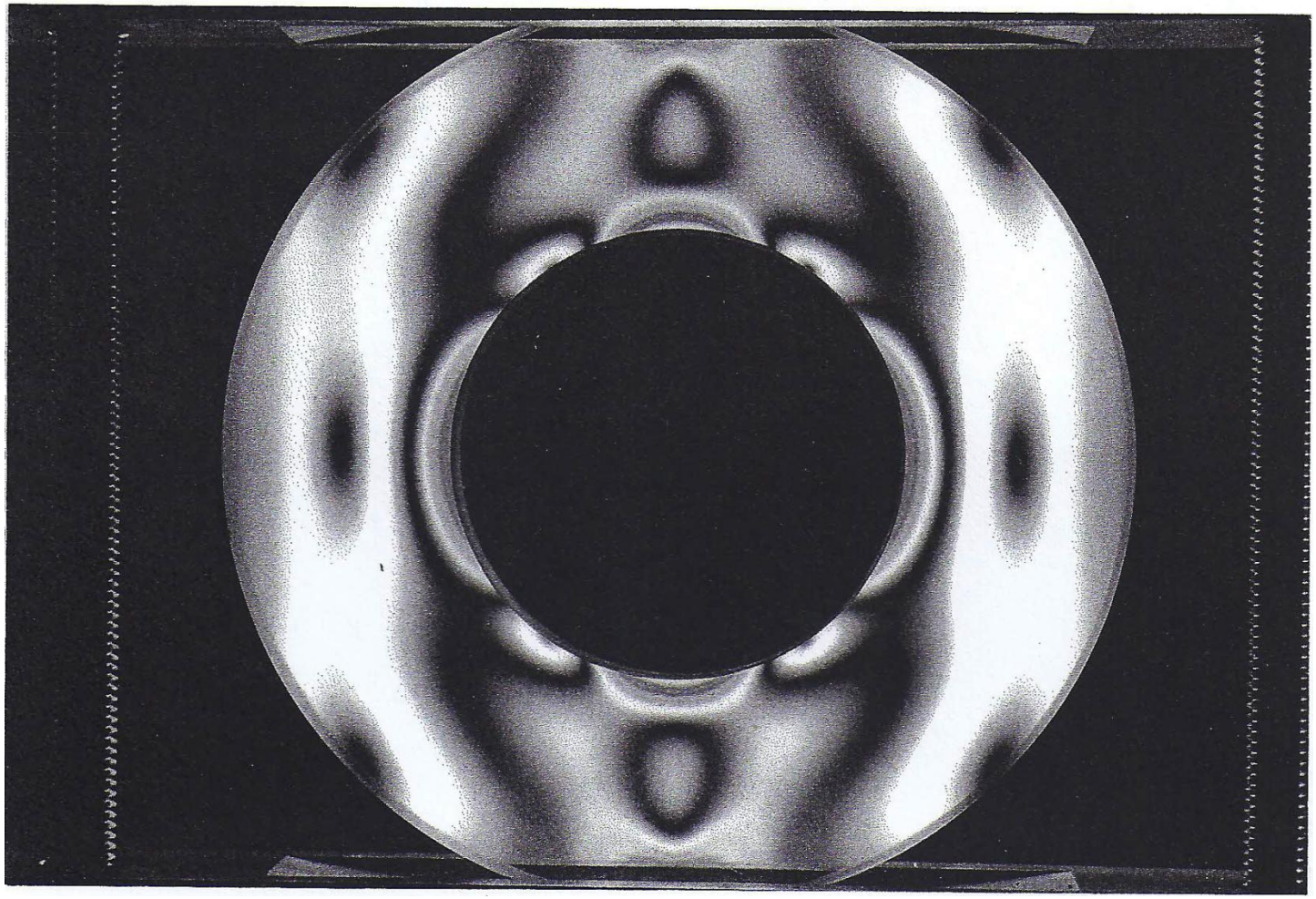


Evolution de la charge en fonction du déplacement

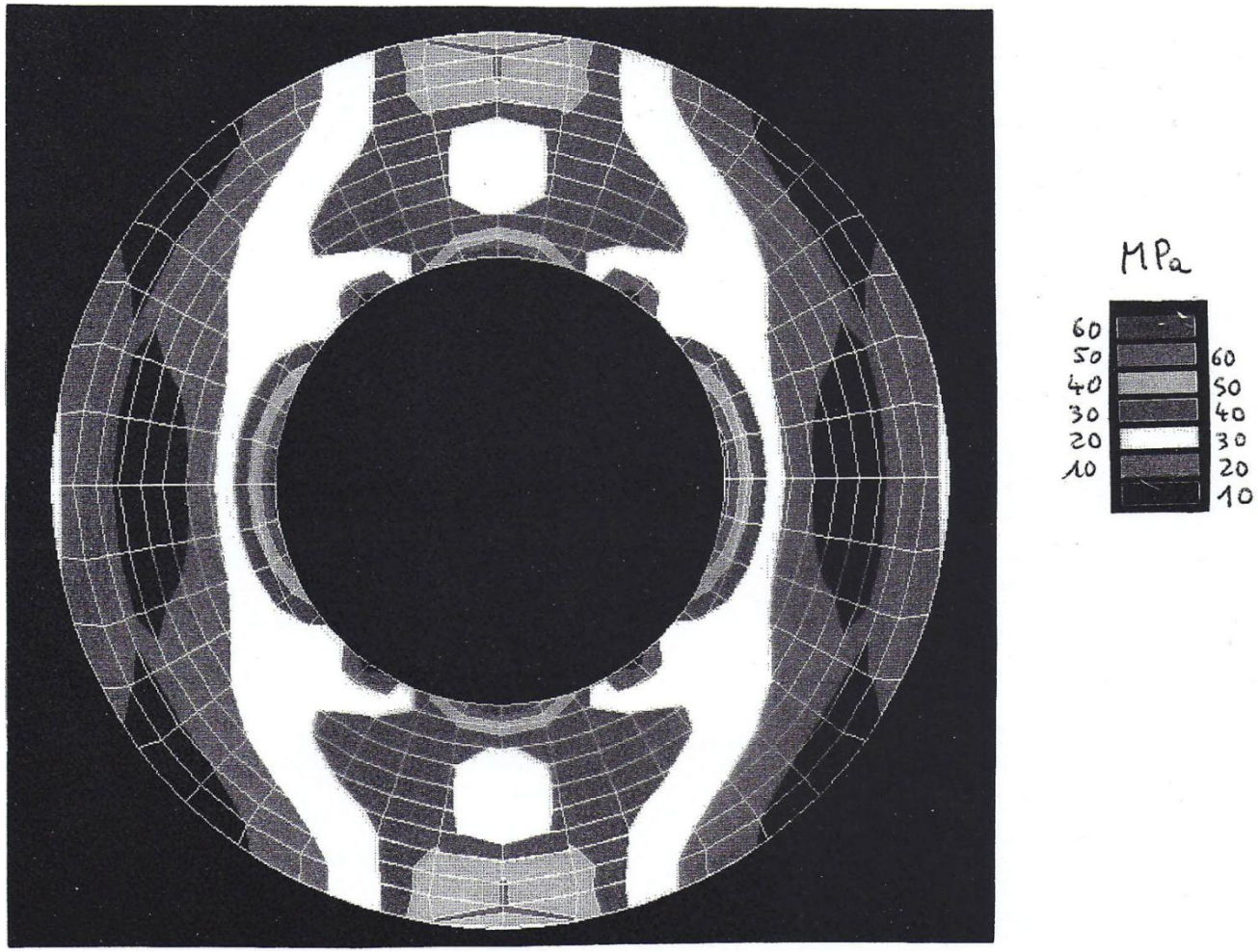


vue de dessus

Dispositif de photoélasticimétrie



Etat de contrainte d'une couronne en cours de compression



Calcul par éléments finis des isovaleurs de contrainte

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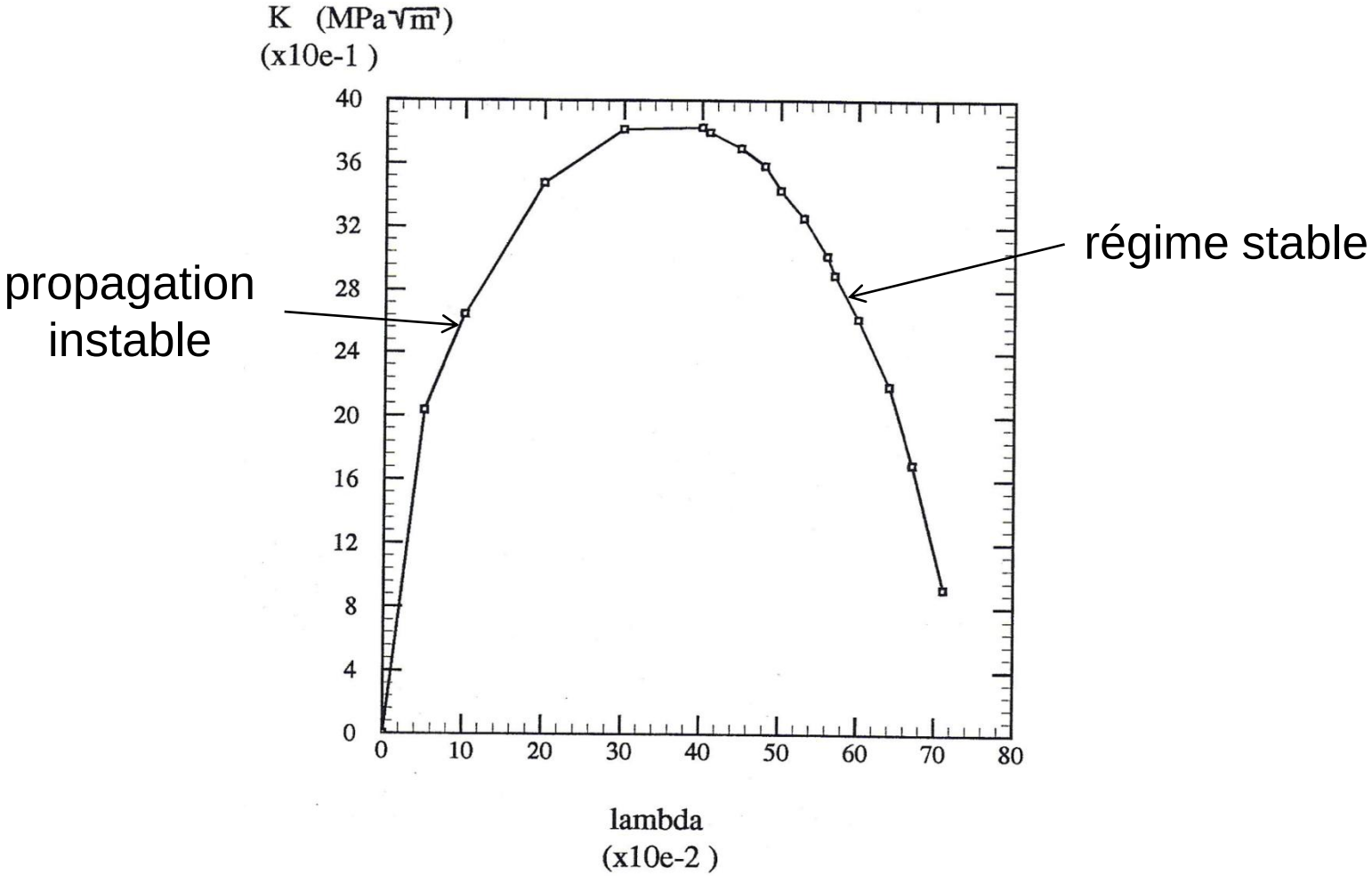
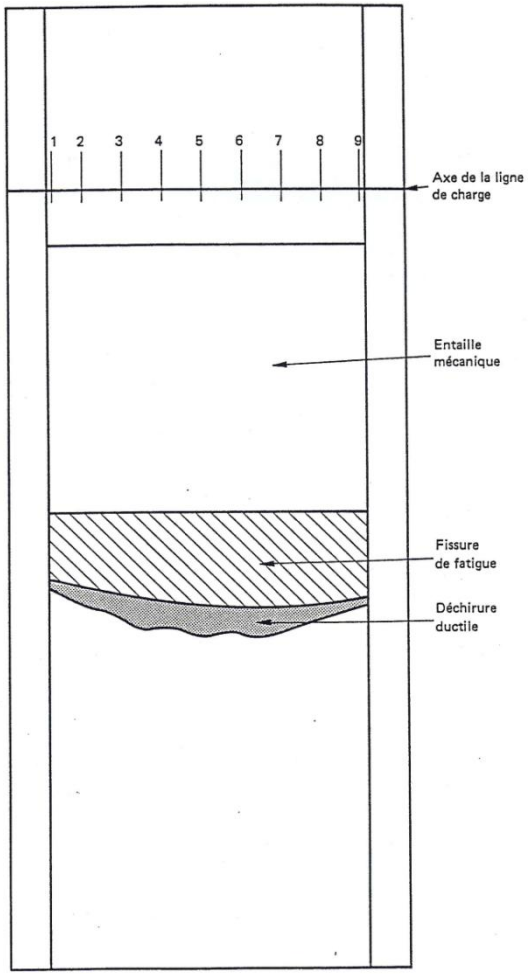


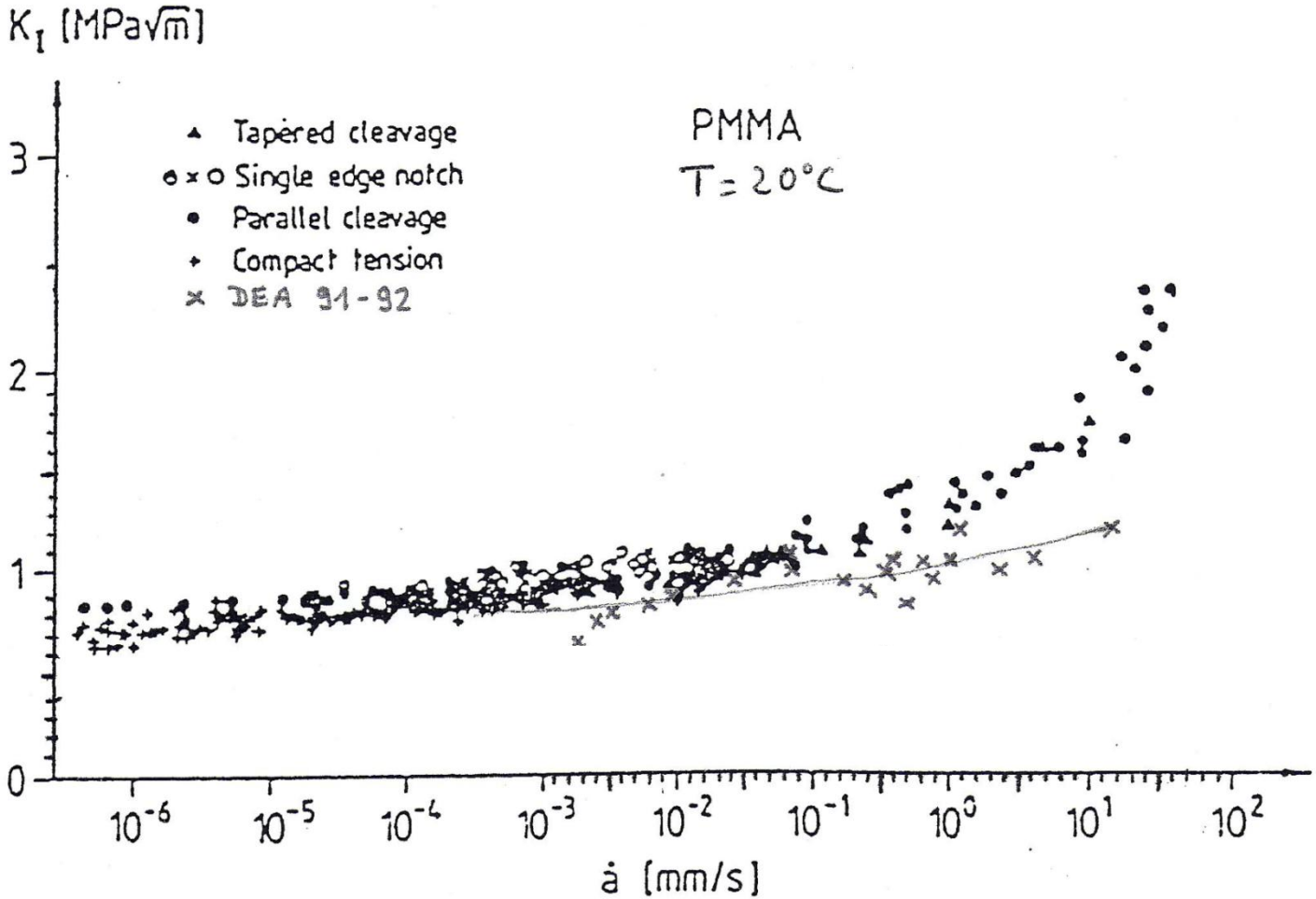
Facteur d'intensité des contraintes en fonction de la fissure

Longueur moyenne de fissure :

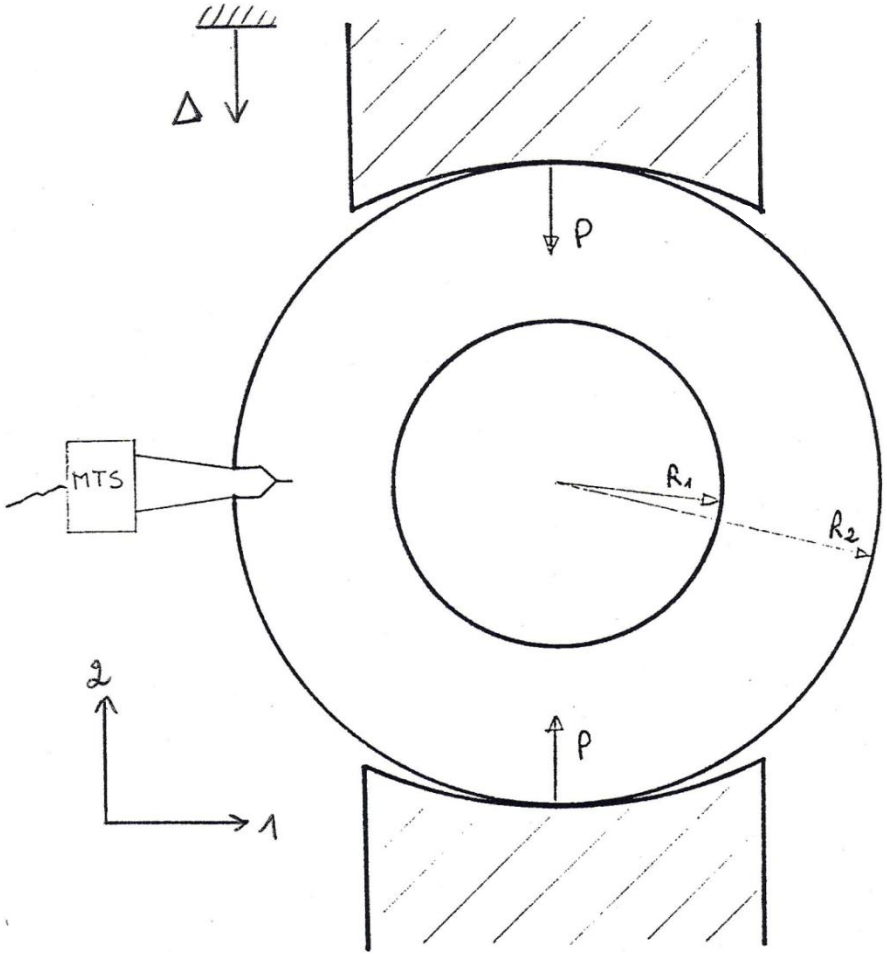
$$a = \frac{1}{8}\left\{\frac{a^1 + a^9}{2} + \sum_{i=2}^{8} a^i\right\}$$



Mesure des longueurs de fissures



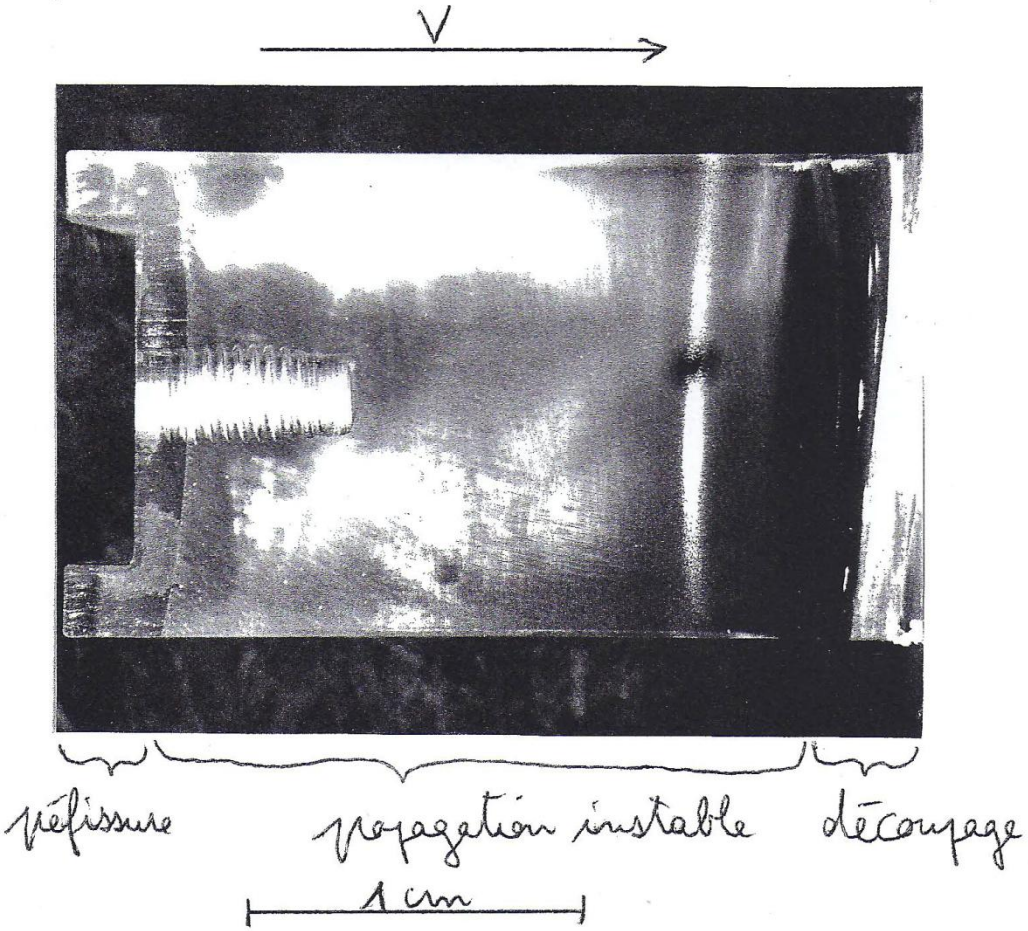
Facteur K_I en fonction de la vitesse de la fissure



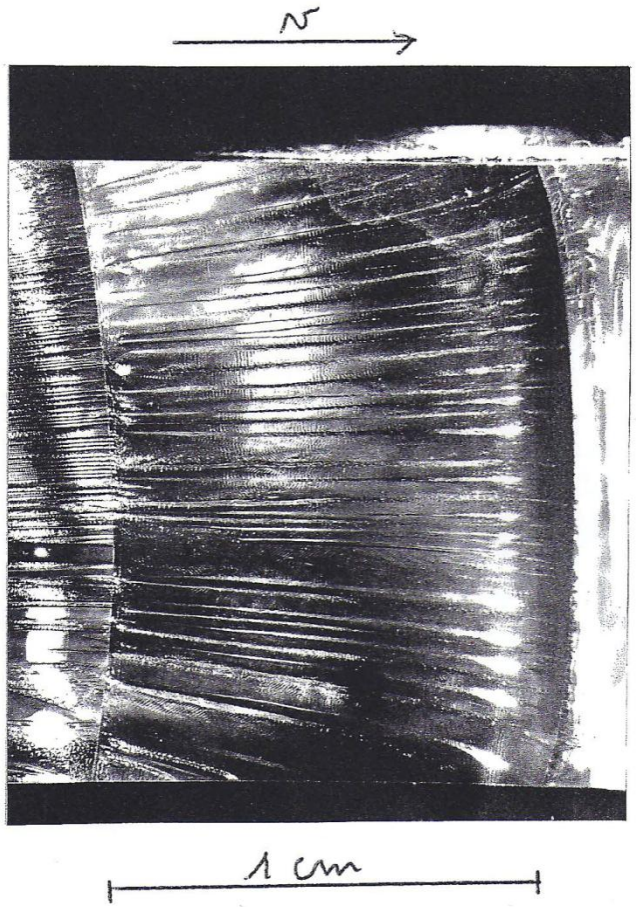
$$\beta = \frac{R_1}{R_2} = \frac{1}{2}$$

$$\lambda = \frac{a}{R_2 - R_1}$$

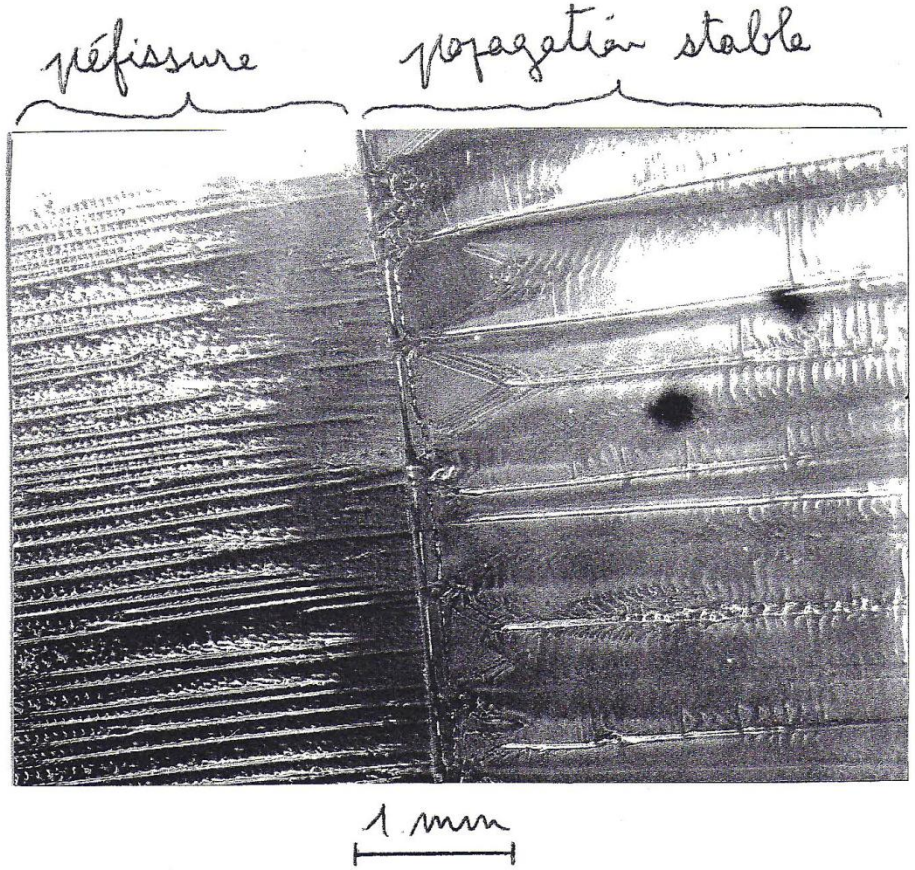
Dispositif expérimental



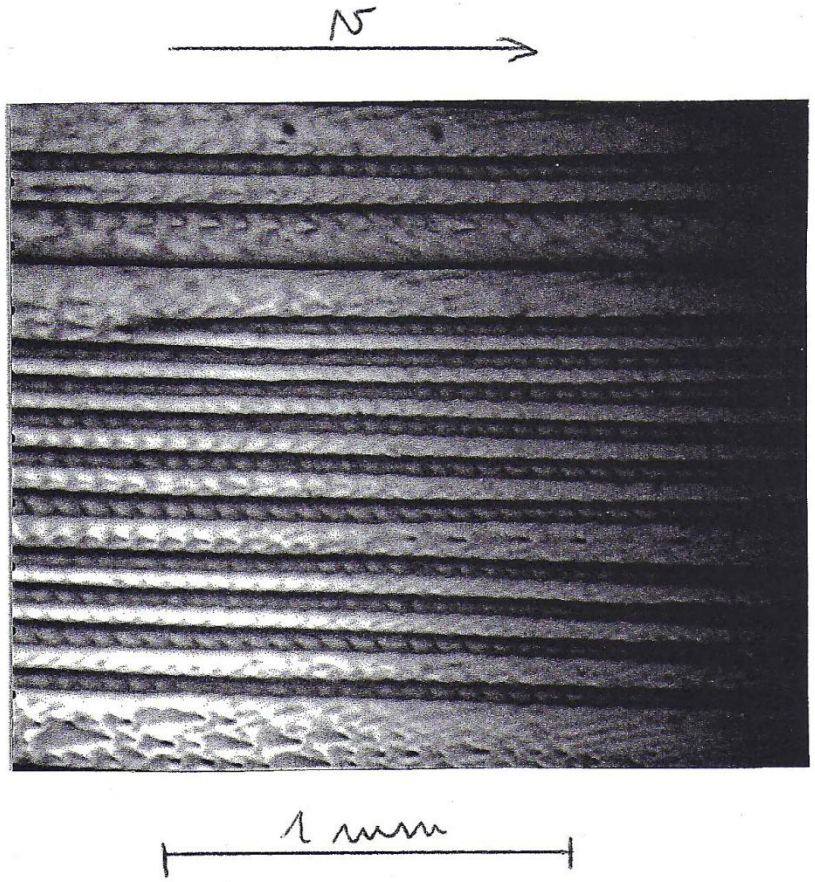
Vue globale du faciès de rupture fragile (binoculaire)



Zone de propagation stable

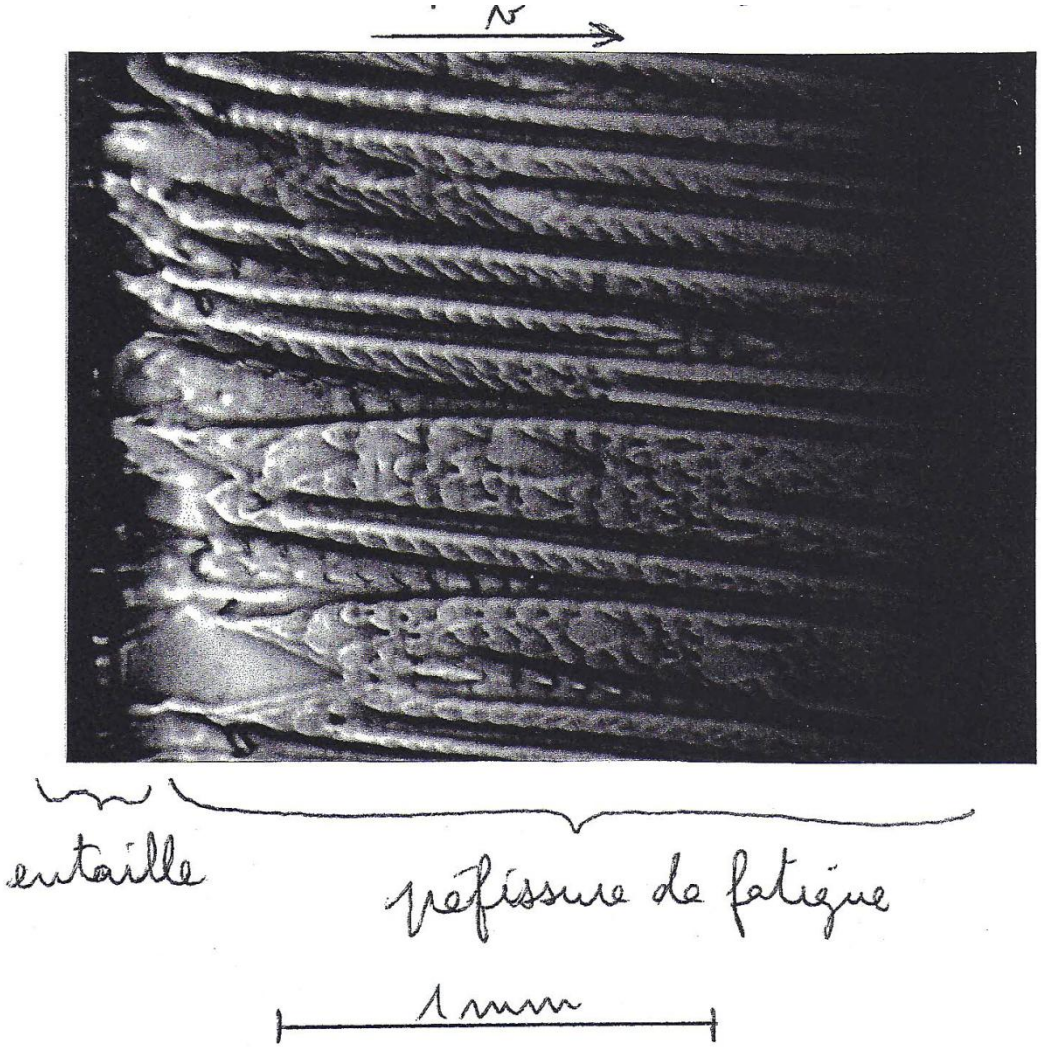


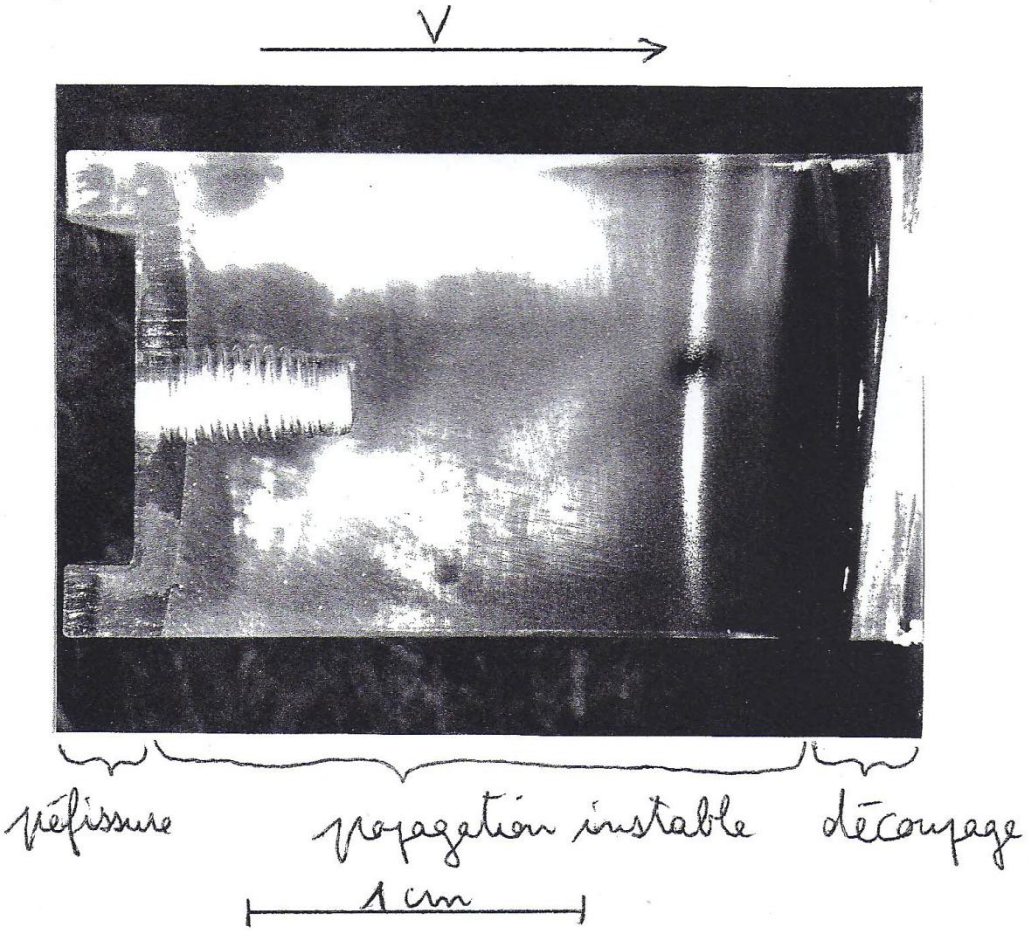
Détail du front de séparation



Stries de préfissuration par fatigue (microscope optique)

II) RESULTATS ET OBSERVATIONS





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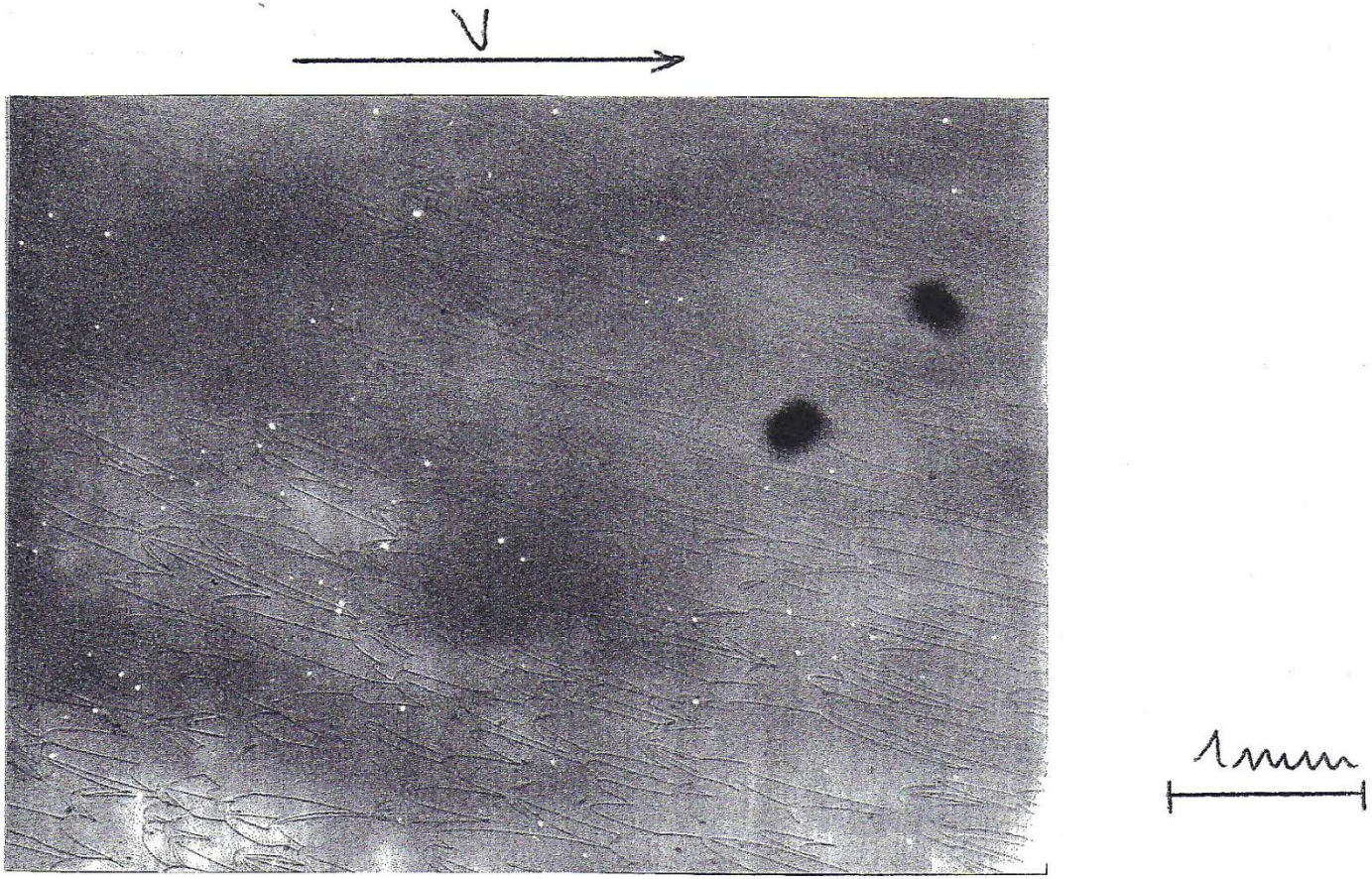
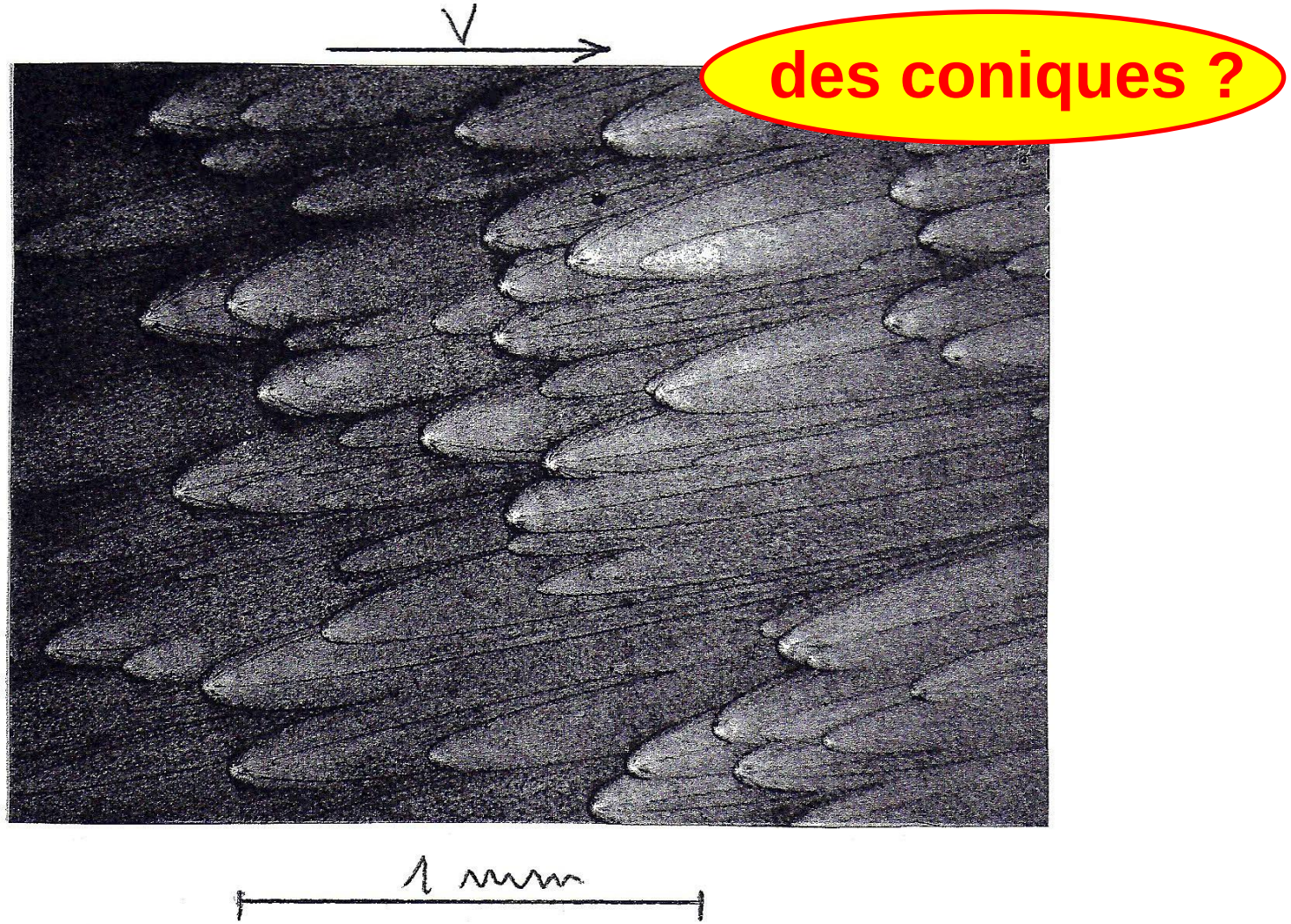


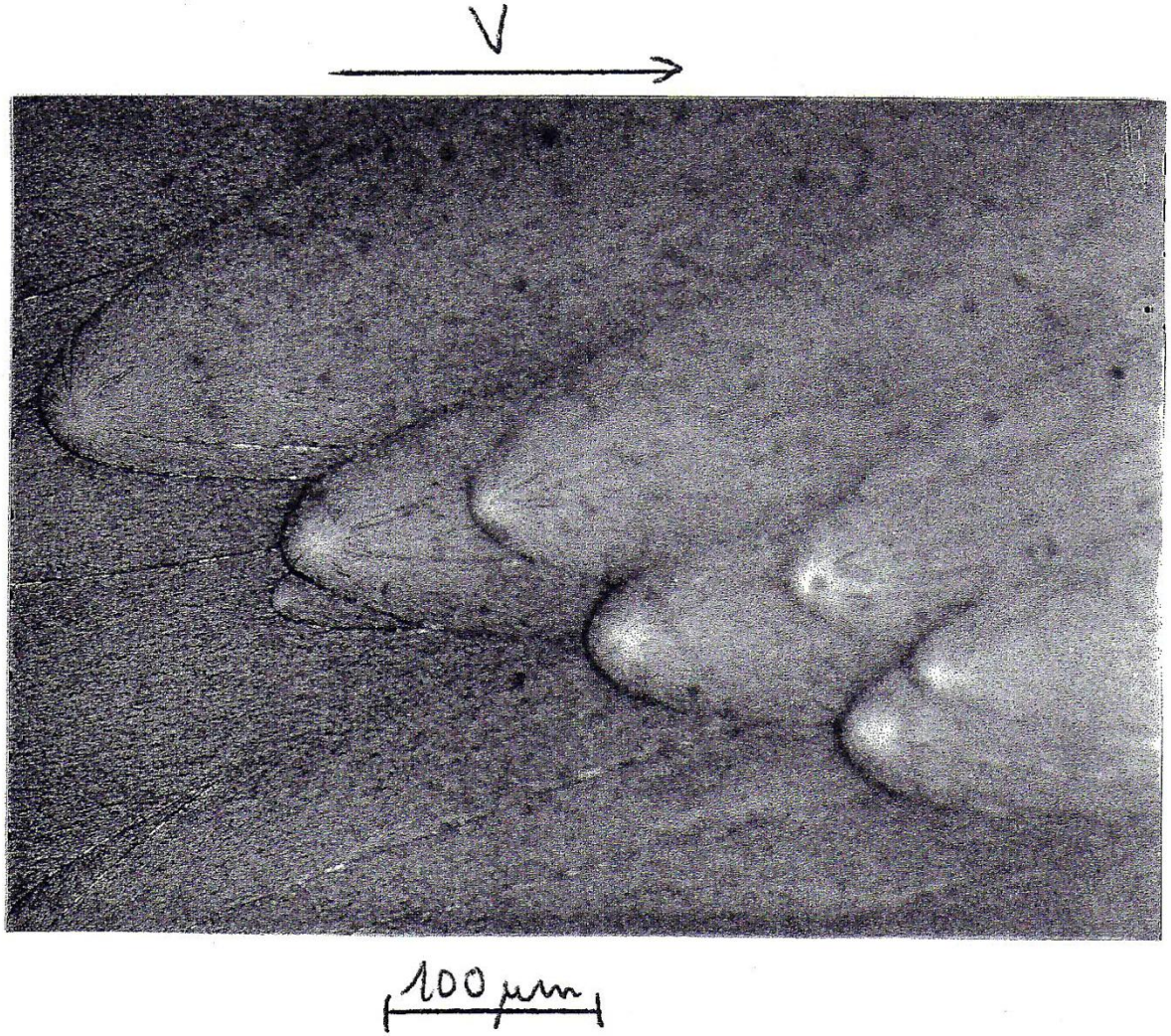
Faciès de propagation instable (microscope optique)

II) RESULTATS ET OBSERVATIONS



des coniques ?

II) RESULTATS ET OBSERVATIONS



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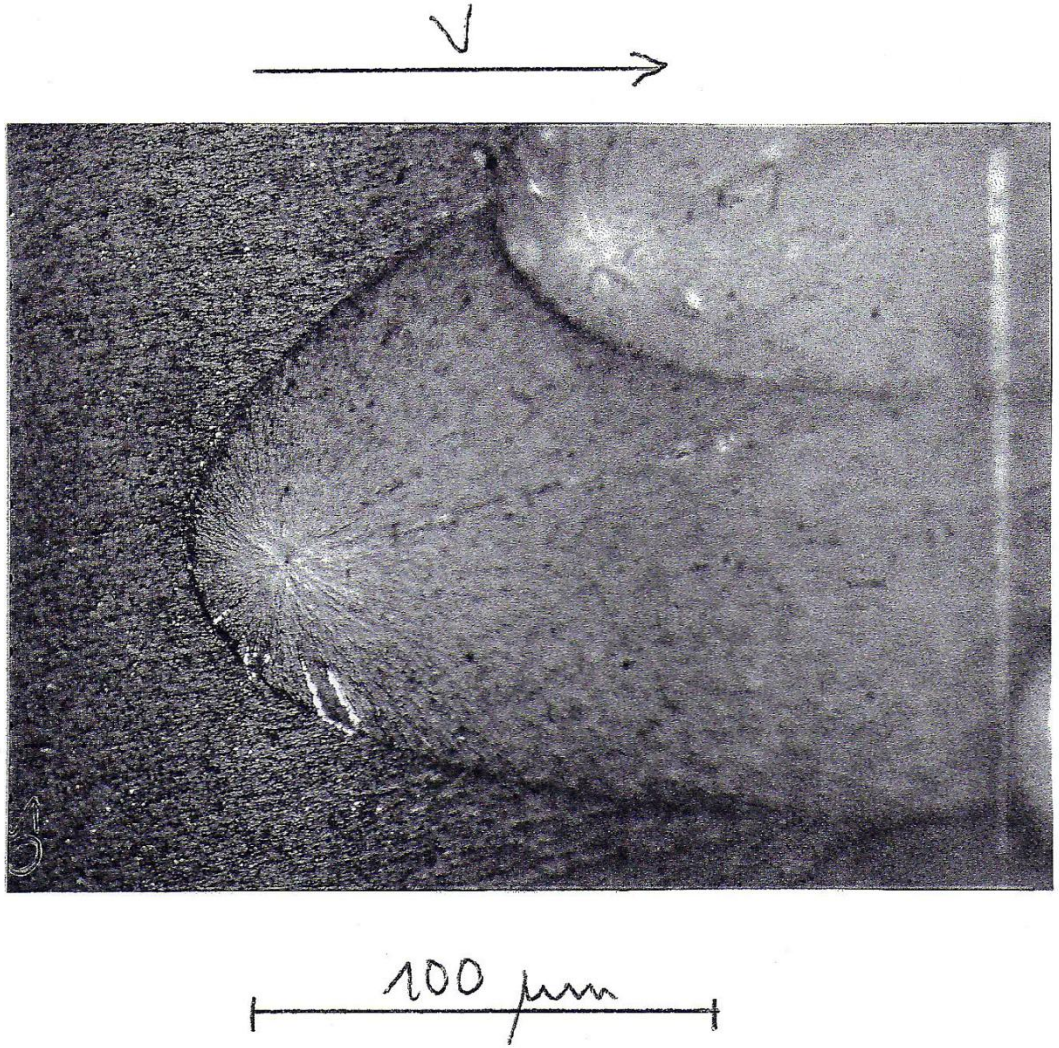

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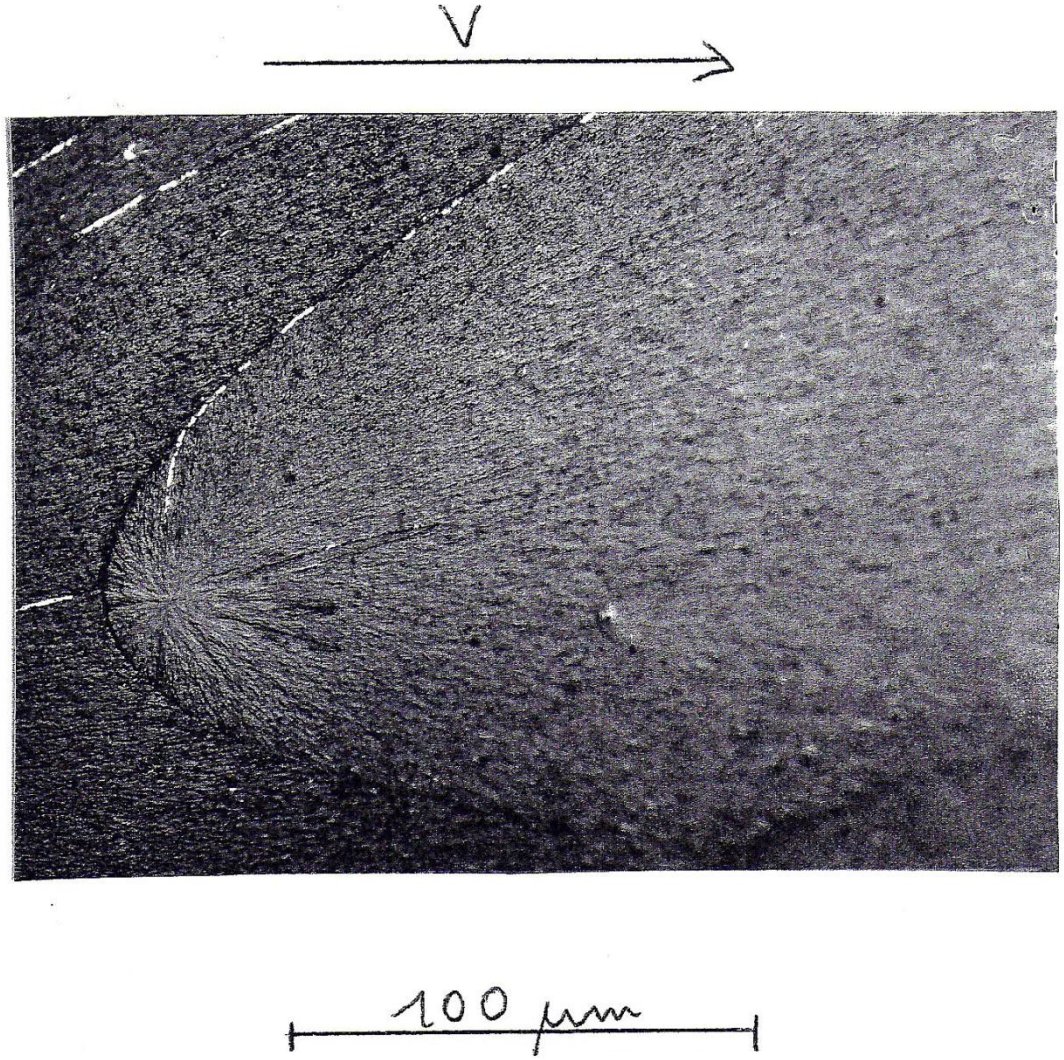

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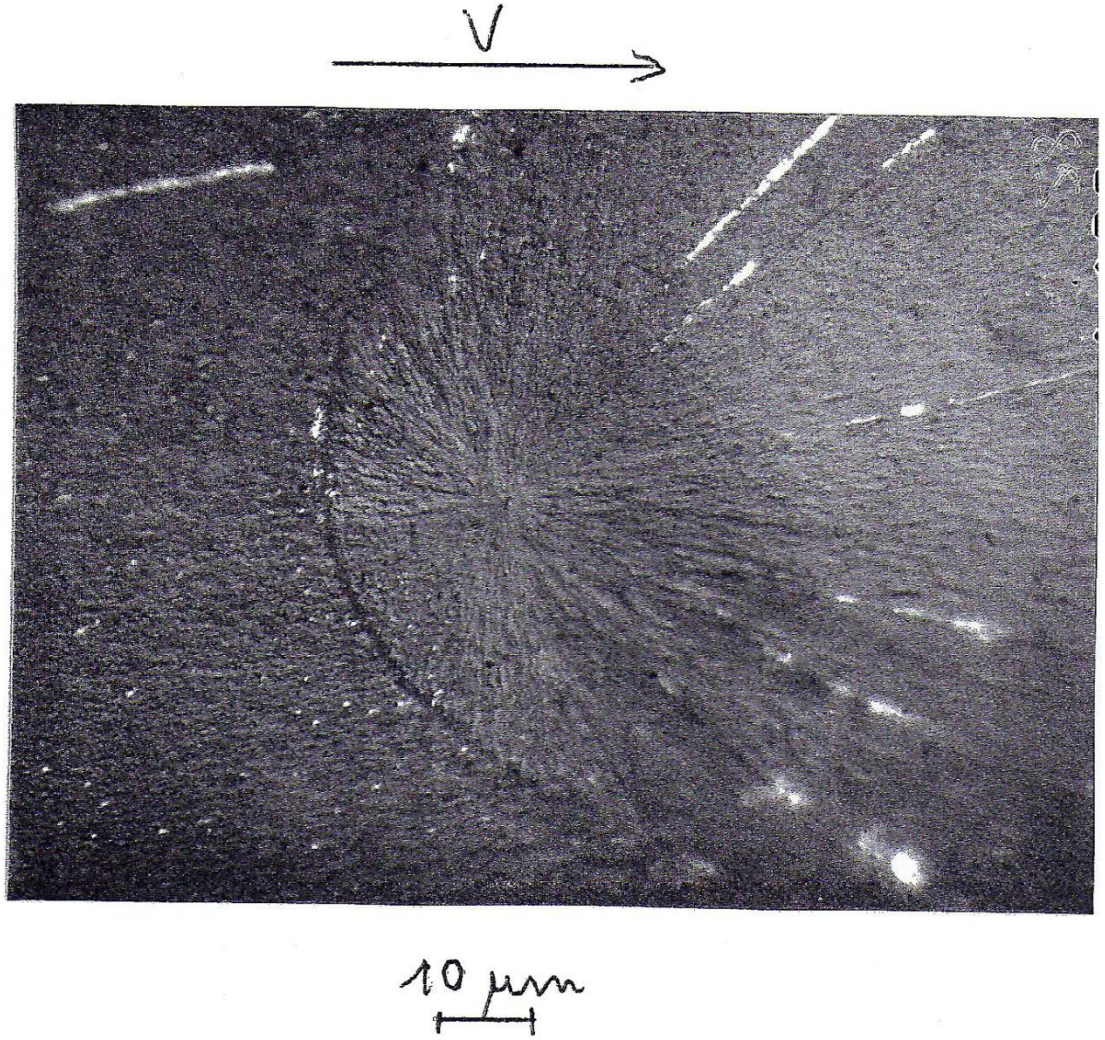
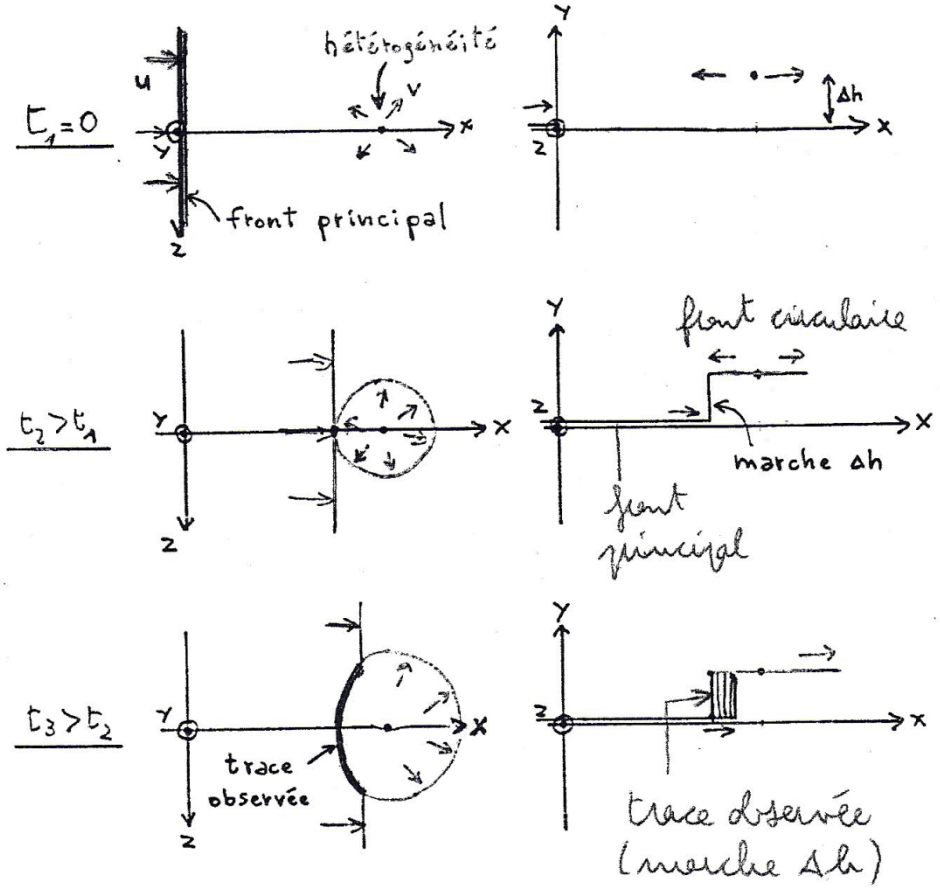
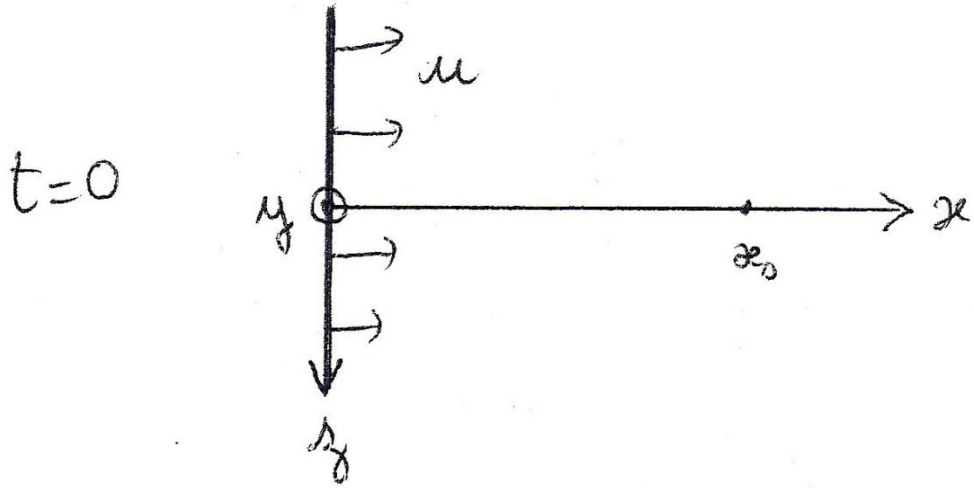


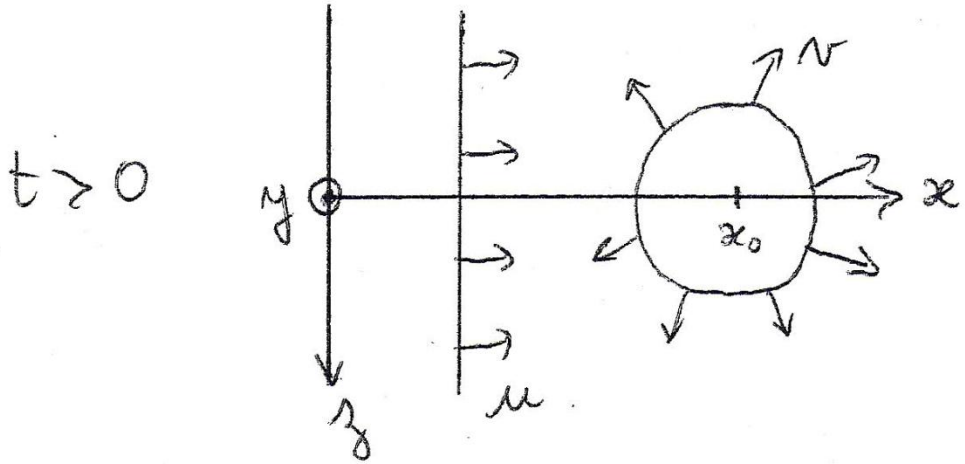
évolution chronologique :

vue de dessus : coupe :



Rencontre de deux fronts de fissuration





Avancée des deux fronts de fissuration

$$* \; x = ut$$

$$* \; (vt)^2 = z^2 + (x - x_0)^2$$

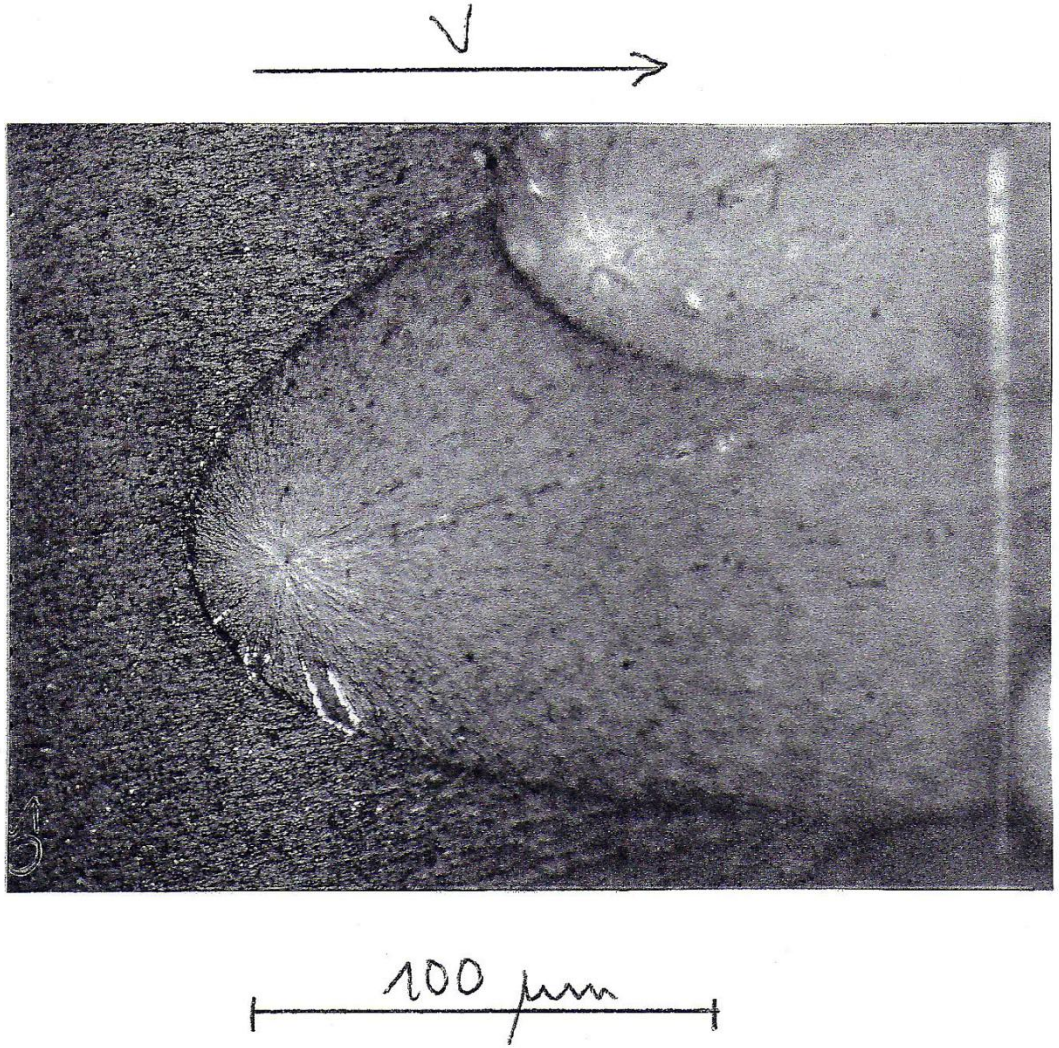
$$d'où$$

$$z^2 + (1 - \frac{v^2}{u^2})x^2 = x_0(2x - x_0)$$

=> des coniques !

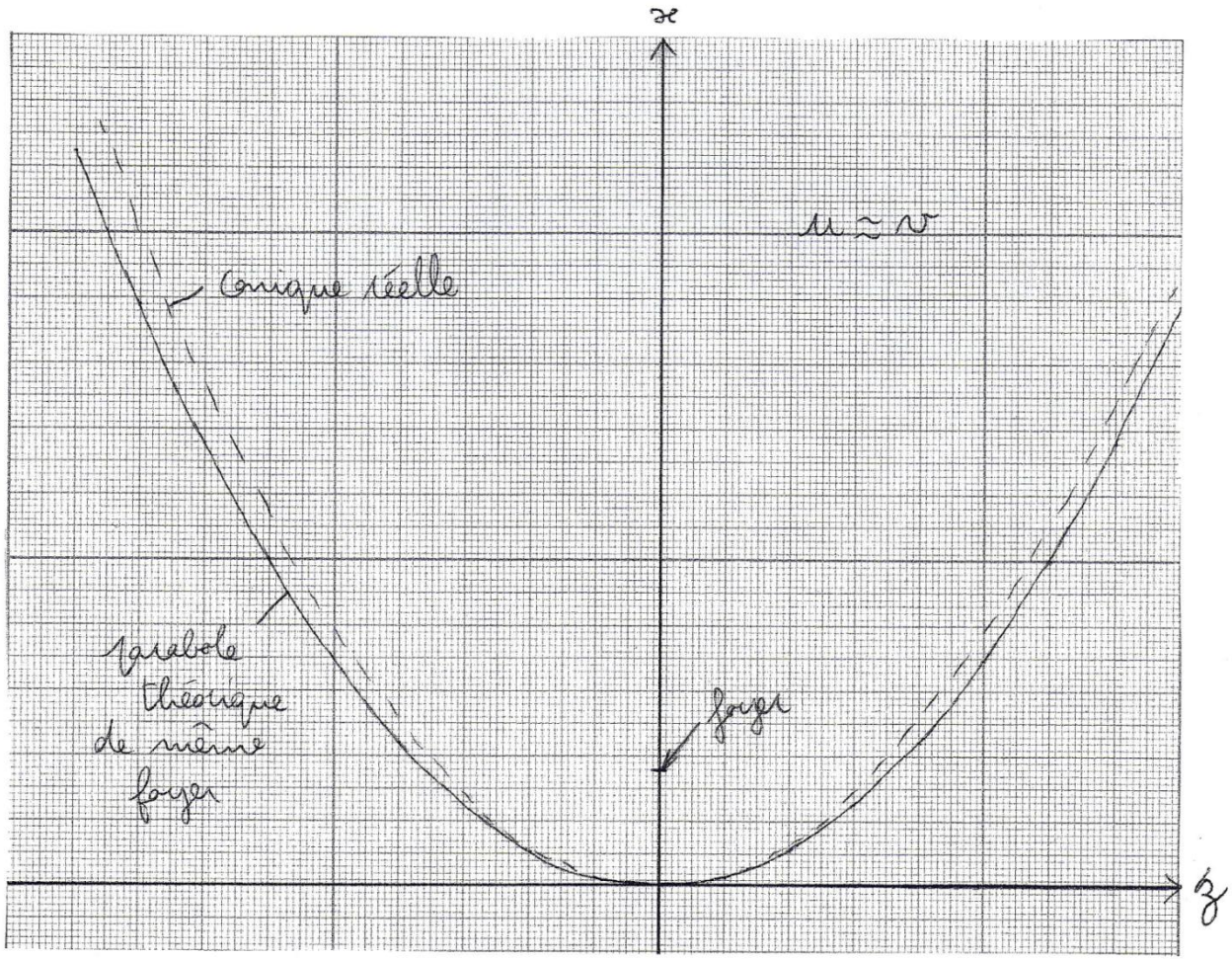
- Si u > v : des ellipses
- Si u = v : des paraboles
- Si u < v : des branches d'hyperboles

II) RESULTATS ET OBSERVATIONS



Si u = v :

$$x = \frac{x_0}{2} + \frac{1}{2x_0} z^2$$

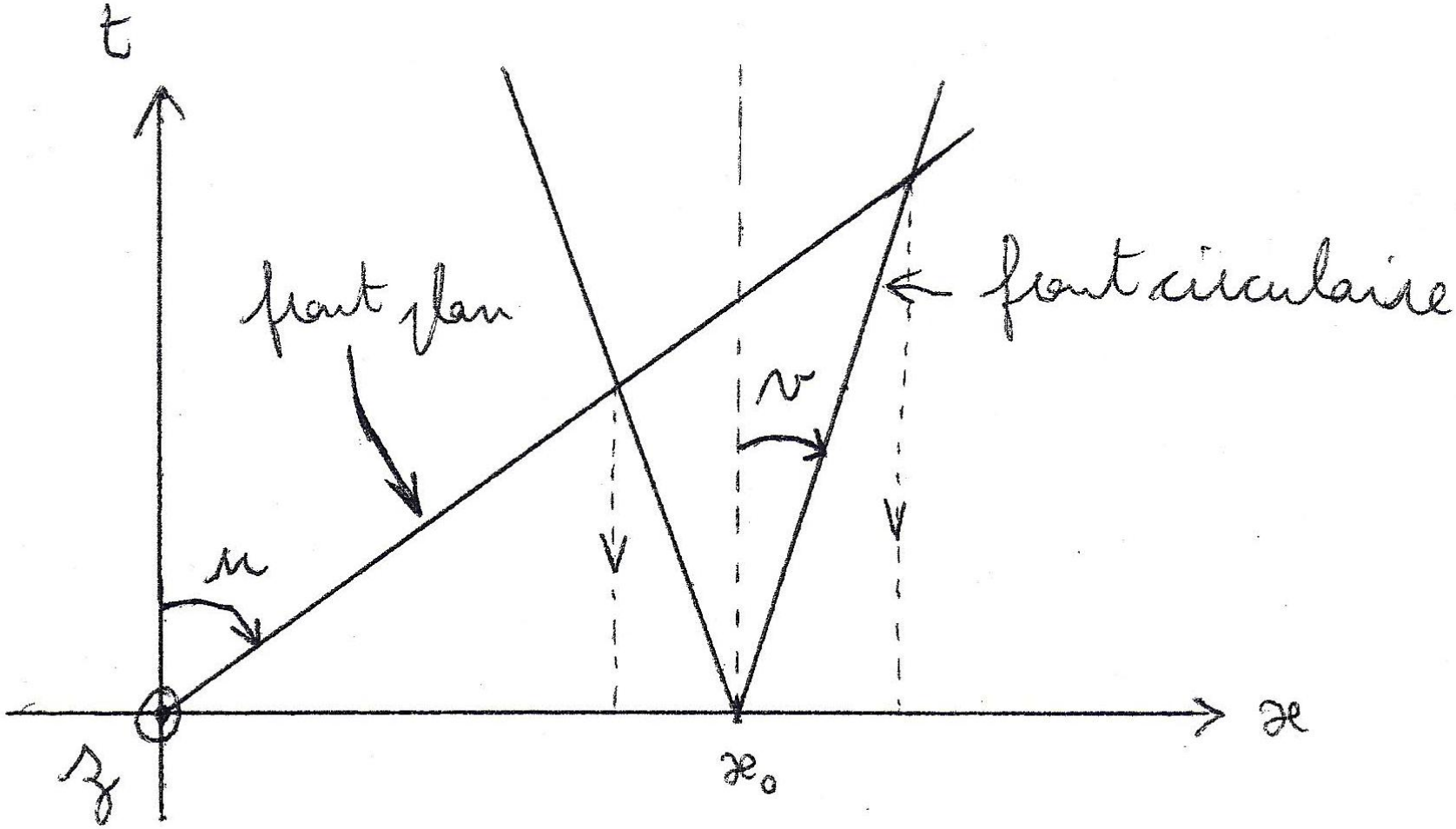


Comparaison entre forme théorique et forme expérimentale

Distances moyennes entre hétérogénéités

=> 50 μm

cf. interstries de fatigue



Evolution des deux fronts de fissure dans un diagramme (x, z, t)

- Si u > v : une ellipse
- Si u = v : une parabole
- Si u < v : une branche d'hyperbole

SYNTHESE

- **Formation de coniques dans un matériau**
- **Modèle phénoménologique**
- **Description mathématique …**

avec approche possible en 3D …

REMERCIEMENTS

- **Professeur André PINEAU, MINES ParisTech**
- **Centre des Matériaux Pierre - Marie Fourt**